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ΣΧΟΛΗ ΜΗΧΑΝΙΚΩΝ

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Πάτρα 2021

ΠΡΟΛΟΓΟΣ

Η παρούσα διπλωματική εργασία εκπονήθηκε στο Πανεπιστήμιο Πελοποννήσου στο τμήμα Μηχανολόγων Μηχανικών. Η μετάφραση του συγκεκριμένου βιβλίου θα διευκολύνει την χρήση του από όλους τους μηχανολόγους καθώς και προπτυχιακούς και μεταπτυχιακούς φοιτητές του τμήματος μηχανολογίας.Επίσης, θεωρούμε ότι θα διευκολύνει και φοιτητές ERASMUS που φοιτούν ή θα φοιτήσουν στο τμήμα αυτό, στο πλαίσιο της κινητικότητας ERASMUS, για σπουδές ή/και για πρακτική άσκηση, οι οποίοι και θα διευκολυνθούν με την μετάφραση του συγκεκριμένου βιβλίου να παρακολουθήσουν με άνεση το σχετικό μάθημα στην αγγλική γλώσσα. Αυτό ακριβώς επιδιώξαμε με την εκπόνηση της συγκεκριμένης διπλωματικής και θέσαμε σαν στόχο όταν αποφασίσαμε να ασχοληθούμε με την μετάφραση του συγκεκριμένου συγγράμματος, αφού είχαμε την συγκατάθεση του διδάσκοντος και συγγραφέα του βιβλίου στην ελληνική γλώσσα,Δρ. Κωσταντίνου Μαυρίδη. Ένας άλλος λόγος που μας οδήγησε στην ανάληψη της συγκεκριμένης διπλωματικής είναι το γεγονός ότι δεν υπάρχει μετάφραση του βιβλίου στην αγγλική γλώσσα.

Κατά την διάρκεια της μετάφρασης ακολουθήσαμε πιστά την διάταξη και τα περιεχόμενα του συγγράμματος. Στην αρχή μελετήσαμε τις γενικές αρχές του υπολογιστικού προγράμματος, στην συνέχεια είδαμε για τις διέπουσες εξισώσεις της ρευστομηχανικής. Επιπρόσθετα, είδαμε την επίλυση εξισώσεων πεπερασμένων διαφορών, ακόμη μελετήσαμε τις διέπουσες εξισώσεις τυρβώδους ροής. Στο πέμπτο κεφάλαιο μελετήσαμε την δομή του υπολογιστικού προγράμματος, ενώ στο έκτο κεφάλαιο μελετήσαμε την ολοκλήρωση των διαφορικών εξισώσεων των μεταβλητών του teach-t και συμβολισμούς τους σε γλώσσα FORTRAN. Στο τελευταίο κεφάλαιο ασχοληθήκαμε με την εφαρμογή του υπολογιστικού προγράμματος.

Τέλος, ευχαριστούμε θερμά την Κυρία Δούσμπη Βασιλική για την πολύτιμη βοήθεια και την ορθή καθοδήγηση που μας προσέφερε, καθώς επίσης ευχαριστούμε και τον Κύριο Μαυρίδη Κωνσταντίνο, ο οποίος μας έδωσε την άδεια να μεταφράσουμε το βιβλίο του στα αγγλικά.

Βλάχος Στέφανος

Ζάτσε Αρτέμης

Φεβρουάριος 2021

Υπεύθυνη Δήλωση Φοιτητών: Οι κάτωθι υπογεγραμμένοι Φοιτητές έχουμε επίγνωση των συνεπειών του Νόμου περί λογοκλοπής και δηλώνουμε υπεύθυνα ότι είμαστε συγγραφείς αυτής της Πτυχιακής Εργασίας, αναλαμβάνοντας την ευθύνη επί ολοκλήρου του κειμένου εξ ίσου, έχουμε δε αναφέρει στην Βιβλιογραφία μας όλες τις πηγές τις οποίες χρησιμοποιήσαμε και λάβαμε ιδέες ή δεδομένα. Δηλώνουμε επίσης ότι, οποιοδήποτε στοιχείο ή κείμενο το οποίο έχουμε ενσωματώσει στην εργασία μας προερχόμενο από Βιβλία ή άλλες εργασίες ή το διαδίκτυο, γραμμένο ακριβώς ή παραφρασμένο, το έχουμε πλήρως αναγνωρίσει ως πνευματικό έργο άλλου συγγραφέα και έχουμε αναφέρει ανελλιπώς το όνομά του και την πηγή προέλευσης.

Οι Φοιτητές

(Ονοματεπώνυμο)   (Ονοματεπώνυμο)

ΒΛΑΧΟΣ ΣΤΕΦΑΝΟΣ   ΑΡΤΕΜΗΣ ΖΑΤΣΕ
....
(Υπογραφή)   (Υπογραφή)

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ΠΕΡΙΛΗΨΗ

Η παρούσα διπλωματική εργασία ξεκίνησε σαν ιδέα λόγω ενδιαφέροντος που υπήρχε για το συγκεκριμένο μάθημα, καθώς και μετά από έρευνα μας διαπιστώσαμε πως δεν υπάρχει μετάφραση του συγγράμματος στην αγγλική γλώσσα, έτσι παίρνοντας την έγκριση του κύριου Μαυρίδη καταλήξαμε στο συγκεκριμένο θέμα.

Το σύγγραμμα αρχικά πραγματεύεται το αντικείμενο και τα χαρακτηριστικά του υπολογιστικού προγράμματος το οποίο είναι γραμμένο για μόνιμη δισδιάστατη, τυρβώδη, επίπεδη, ασυμπίεστη ροή με ανακυκλοφορία. Η επίλυση γίνεται με μια μέθοδο πεπερασμένων διαφορών(hybrid) με τις κύριες μεταβλητές να είναι οι ταχύτητες και η πίεση. Οι διέπουσες εξισώσεις της ρευστομηχανικής κατανέμονται σε 4 κατηγορίες. Είναι οι μερικές διαφορικές εξισώσεις , οι εξισώσεις πεπερασμένων διαφορών , η τελική εξίσωση πεπερασμένων διαφορών και η εξίσωση πεπερασμένων διαφορών για την ορμή.

Η επίλυση εξισώσεων πεπερασμένων διαφορών γίνεται με τη χρήση του αλγόριθμου TDMA. Η επίλυση επιτυγχάνεται με τον υπολογισμό κάθε γραμμής ξεχωριστά. Ο αλγόριθμος simple επιτυγχάνει την επίλυση των εξισώσεων της ορμής έχοντας εκτιμήσει ένα πεδίο πίεσης. Στο τέλος κάθε επανάληψης προσδιορίζεται η σύγκλιση μεθόδου όπου συγκρίνονται οι υπολειπόμενες τιμές κάθε εξίσωσης πεπερασμένων διαφορών με αποτέλεσμα την αποφυγή της απόκλισης. Στην είσοδο της υπολογιστικής περιοχής της ροής η καλή γνώση της κατάστασης βοηθάει στον καθορισμό των μεταβλητών. Ενώ στην έξοδο της υπολογιστικής περιοχής ο προσδιορισμός των μεταβλητών αυτών δεν είναι σημαντικός.

Σε πολλές σύνθετες ροές ενδέχεται να παρουσιασθούν αριθμητικές αστάθειες με αποτέλεσμα την απαίτηση προσθετικών τεχνικών για την επίτευξη της σύγκλισης. Το πρόγραμμα TEACH-T είναι ένα πρόγραμμα που αποτελείται από υπορουτίνες και αναφέρεται σε δισδιάστατες σταθερής κατάστασης για στρωτές η τυρβώδεις ροές. Παρακολουθεί τις επαναλήψεις και κάνει έλεγχο των αποτελεσμάτων δίνοντας στον χρήστη μια καλύτερη αντίληψη της επιτυχίας η αποτυχίας της υπολογιστικής διαδικασίας. Οι διάφορες εξισώσεις μετατρέπονται σε πεπερασμένες και οι μεταβλητές συμβολίζονται σε γλώσσα προγραμματισμού. Η αριθμητική επίλυση των εξισώσεων στο TEACH γίνεται με την μετατροπή τους σε εξισώσεις πεπερασμένων διαφορών με την μέθοδο των όγκων ελέγχου.

Για τον υπολογισμό της πίεσης εφαρμόζεται μια ειδική τεχνική, με την βοήθεια της εξίσωσης της συνέχειας η οποία δεν έχει όρους πίεσης αλλά μόνο ταχύτητας. Στην εφαρμογή του υπολογιστικού προγράμματος το πρώτο βήμα

για ένα ειδικό πρόβλημα είναι ο καθορισμός της περιοχής επίλυσης και του πλέγματος. Η περιοχή επίλυσης περιορίζεται από το επίπεδο εισόδου του άξονα συμμετρίας και το επίπεδο εξόδου.

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ΕΙΣΑΓΩΓΗ

Τα τελευταία είκοσι χρόνια, η αριθμητική επίλυση των προβλημάτων της ρευστοδυναμικής έγινε πολύ ελκυστική και ένας μεγάλος αριθμός επιστημόνων έχει ασχοληθεί με το σχετικό θέμα.

Για την επίλυση οποιουδήποτε πεδίου ροής, είναι πλέον δυνατή η χρήση υπολογιστικών πακέτων, τα οποία, μετά την προσαρμογή του υπό εξέταση προβλήματος, δίνουν μια αριθμητική λύση που προσεγγίζει την πραγματική. Ένα πρόβλημα υπολογιστικής ρευστής μηχανικής, σε μια προσπάθεια επίτευξης του πιο ρεαλιστικού δυνατού αποτελέσματος, πρέπει να πληροί ορισμένα γενικά χαρακτηριστικά.

Τα βασικά στοιχεία είναι τα ακόλουθα:

- Η δημιουργία ενός πεπερασμένου πίνακα σημείων (κόμβων) του πεδίου ροής, που αποτελούν το υπολογιστικό πλέγμα.
- Η μετατροπή των μερικών διαφορικών εξισώσεων σε ένα σύστημα παρόμοιων αλγεβρικών εξισώσεων πεπερασμένων διαφορών, οι οποίες συσχετίζουν τις τιμές των αντίστοιχων μεταβλητών στους κόμβους του πεδίου με τις τιμές των γειτονικών κόμβων.
- Η μετατροπή των εξισώσεων σε κατάλληλη μορφή για την επίλυσή τους με κάποια αξιόπιστη επαναληπτική τεχνική.
- Η παρουσίαση και αξιολόγηση των αποτελεσμάτων της επαναληπτικής διαδικασίας.

Επίσης, οι όροι που ακολουθούν αυξάνουν την αξιοπιστία της υπολογιστικής λύσης:

- **Γενικότητα εφαρμογής**: Μια προσπάθεια δημιουργίας υπολογιστικού κώδικα με πεδίο εφαρμογής σε όσο το δυνατόν περισσότερες περιπτώσεις ροής, με τις λιγότερες δυνατές αλλαγές.
- **Ακρίβεια λύσης**: Οι αλγεβρικές εξισώσεις είναι προσεγγίσεις μερικών διαφορικών εξισώσεων. Η διαφορά μεταξύ των αριθμητικών λύσεων του πρώτου και των αναλυτικών λύσεων του τελευταίου, δηλαδή του υπολογιστικού σφάλματος (trancation error), πρέπει να είναι όσο το δυνατόν μικρότερη.
- **Σύγκλιση**: Ένα χαρακτηριστικό μιας επαναληπτικής μεθόδου επίλυσης ενός συστήματος παρόμοιων αλγεβρικών εξισώσεων που οδηγεί στην ομαλή λύση τους. Μια αξιόπιστη μέθοδος συγκλίνει υπό οποιεσδήποτε συνθήκες.
- **Λύση Οικονομία**: Η βασική ανάγκη μιας μεθόδου είναι να είναι γρήγορη και οικονομική. Αυτή η ανάγκη για «υπολογιστική

οικονομία» παίζει σημαντικό ρόλο στη δημιουργία οποιασδήποτε υπολογιστικής μεθόδου.

Οι προβλέψεις για τη μεταφορά θερμότητας και τη μηχανική ρευστών μπορούν να ληφθούν με δύο κύριες μεθόδους: την πειραματική έρευνα και τον θεωρητικό υπολογισμό. Τα τελευταία χρόνια, η συνεχής αύξηση της χωρητικότητας των υπολογιστών, η βελτίωση των μεθόδων επίλυσης των εξισώσεων Navier-Stokes (ή Reynolds) και η βελτίωση των τυρβώδη μοντέλων που χρησιμοποιούνται για τον υπολογισμό των τυρβώδους τάσεων, που εμφανίζονται στις εξισώσεις Reynolds, σε συνδυασμό με την ανάγκη για χαμηλού κόστους μεθόδους σχεδιασμού, έχουν οδηγήσει στην εκτεταμένη χρήση της Υπολογιστικής Θερμο-Ρευστικής Μηχανικής (Υπολογιστική Ρευστική Δυναμική και Μεταφορά Θερμότητας (CFDHT)) ως ένα επιπλέον εργαλείο διεξαγωγής πειραμάτων καθώς και της διαδικασίας σχεδιασμού.

Τα πλεονεκτήματα του θεωρητικού υπολογισμού σε σχέση με την αντίστοιχη πειραματική έρευνα είναι:

- **Το χαμηλό κόστος**, το πιο σημαντικό πλεονέκτημα μιας υπολογιστικής πρόβλεψης. Στις περισσότερες εφαρμογές, το κόστος κατασκευής και εκτέλεσης ενός υπολογιστικού προγράμματος είναι πολλές φορές χαμηλότερο από το κόστος μιας αντίστοιχης πειραματικής έρευνας.
- **Η ταχύτητα** κατασκευής και εκτέλεσης υπολογιστικής έρευνας σε αντίθεση με αντίστοιχη πειραματική έρευνα.
- **Οι πλήρεις πληροφορίες** που συλλαμβάνονται (με την επίλυση ενός προβλήματος με υπολογιστική μέθοδο) σε ολόκληρη την επιφάνεια της λύσης που μας ενδιαφέρει. Σε αντίθεση με το πείραμα, υπάρχουν ελάχιστες απρόσιτες υπολογιστικές περιοχές και δεν υπάρχει επίσης διαταραχή ροής από τα όργανα μέτρησης.
- **Η δυνατότητα εύκολης προσομοίωσης** σε έναν θεωρητικό υπολογισμό των πραγματικών συνθηκών. Δεν είναι δύσκολο για ένα υπολογιστικό πρόγραμμα να μιμείται μικρές ή μεγάλες διαστάσεις, να χειρίζεται χαμηλές ή υψηλές θερμοκρασίες, τοξικές ή εύφλεκτες ουσίες ή να ακολουθεί πολύ γρήγορες ή πολύ αργές διαδικασίες.
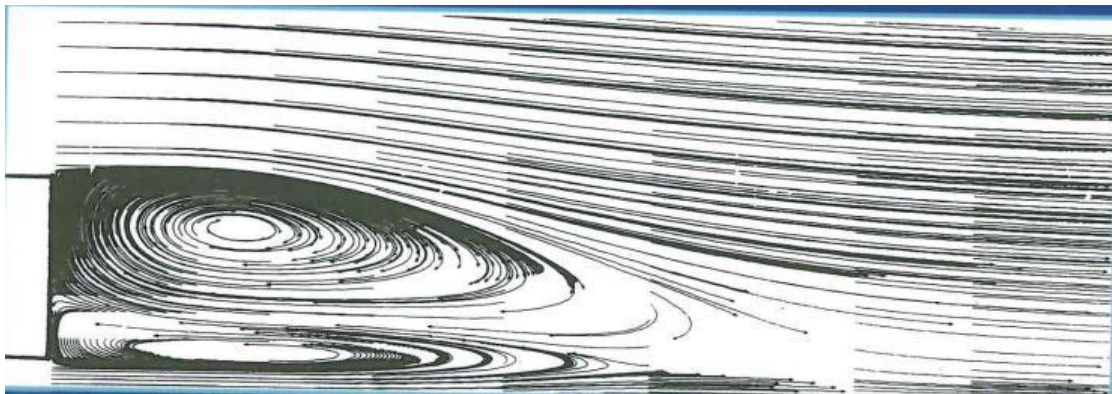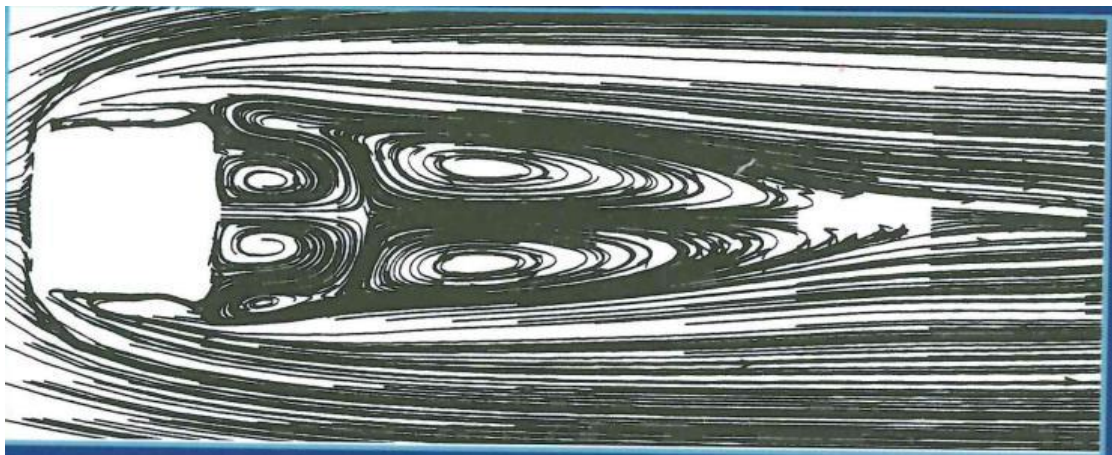
Παρά τα παραπάνω πλεονεκτήματα του θεωρητικού υπολογισμού, δεν πρέπει να υπάρχει υπερβολικός ενθουσιασμός γι 'αυτό και είναι χρήσιμο να ληφθούν υπόψη και τα μειονεκτήματα και οι περιορισμοί του. Ο υπολογισμός με τη χρήση υπολογιστών επιλύει τις συνέπειες ενός μαθηματικού μοντέλου. Η πειραματική έρευνα, από την άλλη πλευρά, παρατηρεί την πραγματικότητα.

Τα όρια ισχύος ενός μαθηματικού μοντέλου περιορίζουν τη χρησιμότητα του θεωρητικού υπολογισμού. Το τελικό αποτέλεσμα του θεωρητικού υπολογισμού εξαρτάται τόσο από το μαθηματικό μοντέλο όσο και από την αριθμητική μέθοδο που χρησιμοποιείται. Μια σωστή προσπάθεια για μια πρόβλεψη πρέπει να είναι ένας συνετός συνδυασμός θεωρητικού υπολογισμού και πειράματος. Η εμπλοκή των δύο παραγόντων εξαρτάται από τη φύση του προς επίλυση προβλήματος, τους στόχους της πρόβλεψης και τους οικονομικούς και άλλους περιορισμούς της κατάστασης.

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Dr. of MECHANICAL ENGINEERING & AERONAUTICAL ENGINEER

COMPUTATIONAL
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ION PUBLICATIONS

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IMPLEMENTATION OF THE COMPUTATIONAL PROGRAM

INTRODUCTION

In the last twenty years the numerical resolution of Fluid Dynamics Problems has become very attractive and a large number of scientists have dealt with the relevant subject.

In order to resolve any flow field, it is now possible to use computational packages, which, after the adaptation of the problem under investigation, give a numerical solution that approximates the actual one. A Computational Fluid Mechanics problem, in an effort to achieve the most realistic possible result, must meet some certain general characteristics.

Its main elements are the following:

- The creation of a finite table of points (nodes) of the flow field, which make up the computational grid.
- The conversion of the partial differential equations into a system of similar algebraic finite difference equations, which correlate the values of the respective variables at the nodes of the field with the values of the neighboring nodes.
- The transformation of equations into a suitable form for their resolution by some reliable iterative technique.
- The presentation and evaluation of the results of the iterative process.

Also, the following terms increase the reliability of the computational solution:

- **Generality of implementation:** An attempt to create a computational code with scope in as many flow cases as possible, with the fewest possible changes.
- **Solution Accuracy:** Algebraic equations are approximations of partial differential equations. The difference between the numerical solutions of the former and the analytical solutions of the latter, namely the computational error (trancation error), should be as small as possible.
- **Convergence:** A feature of an iterative method of resolving a system of similar algebraic equations that leads to their smooth solution. A reliable method converges under any conditions.
- **Solution Economy:** The basic need of a method is to be quick and economical. This need for "computational economy" plays a major role in the creation of any computational method.

Predictions about Heat Transfer and Fluid Mechanics can be obtained by two main methods: the experimental research and the theoretical calculation. In recent years, the continuous increase in the capacity of computers, the

improvement of the methods for resolving the Navier-Stokes (or Reynolds) equations and the improvement of the turbulent models used to calculate turbulent stresses, which appear in the Reynolds equations, combined with the need for low-cost design methods, have led to the extensive use of the Computational Thermo-Fluid Mechanics (Computational Fluid Dynamics and Heat Transfer (CFDHT)) as an additional tool of conducting experiments as well as the design process.

The advantages of the theoretical calculation over the corresponding experimental research are:

- **The low cost,** the most important advantage of a computational prediction. In most applications, the cost of building and running a computational program is many times lower than the cost of a corresponding experimental investigation.
- **The speed** of building and executing a computational investigation as opposed to a corresponding experimental investigation.
- **The complete information** that is captured (with the solution of a problem by a computational method) in the whole solution surface that we are interested in. In contrast to the experiment, there are minimal inaccessible computational areas and there is also no flow disturbance from the measuring instruments.
- **The possibility of easy simulation** in a theoretical calculation of real conditions. It is not difficult for a computational program to emulate small or large dimensions, to handle low or high temperatures, toxic or flammable substances or to follow very fast or very slow processes.

Despite the above advantages of the theoretical calculation, there should be no extreme enthusiasm about it and it is useful to take into account its disadvantages and limitations as well. The calculation with the use of Computers resolves the consequences of a mathematical model. The experimental research, on the other hand, observes reality. The power limits of a mathematical model restrain the usefulness of the theoretical calculation. The final result of the theoretical calculation depends on both the mathematical model and the numerical method used. A correct attempt for a prediction should be a prudent combination of theoretical calculation and experiment. The involvement of the two factors depends on the nature of the problem to be solved, the objectives of the prediction and the financial and other restrictions of the situation.

CHAPTER 1

GENERAL PRINCIPLES OF A COMPUTATIONAL PROGRAM

1.1 Object and Features of a Computational Program

Chapters 1 to 7 give the key features of the general prediction method for the transfer of momentum, mass and heat, which are integrated in a computational code. The computational program is written for constant two-dimensional (expandable to three-dimensional), turbulent (or laminar), flat (or axis-symmetric), uncompressed flow with recirculation. There is a possibility for extension in order to be applied to non-constant three-dimensional flows. The provided program list is written for flow in a tube with abrupt expansion (STEP geometry).

The computational code resolves the relative equations of conservation of momentum, mass, energy and so on, with a finite difference method **(hybrid)**. The main hydrodynamic variables used are the speeds and the pressure. A special procedure, the **SIMPLE** method (Patankar and Spalding,1972), is used to solve the speed and pressure fields, and each equation is resolved with an **LBL** resolving process, using the **TDMA** algorithm.

1.2 Method Applications

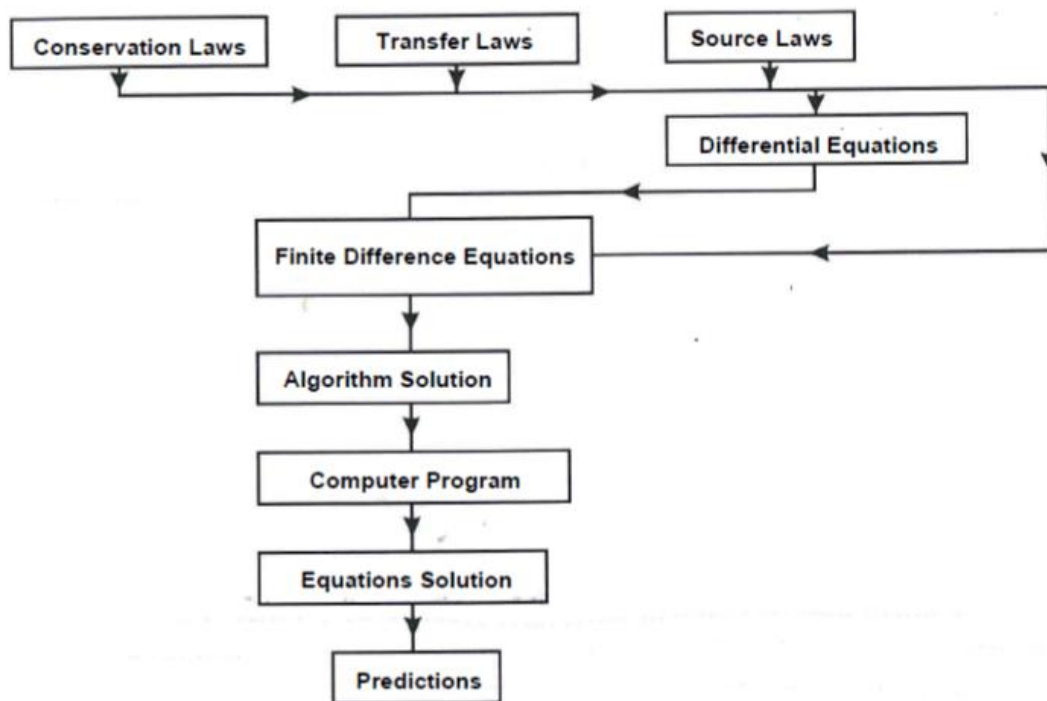In various fields the method is applied in the following fields:

- **In the power generation field:** gas turbines, reciprocating engines, burners and nuclear reactors.
- **In chemical plants:** heat exchangers, blast furnaces.
- **In environmental studies:** pollutants prediction, disposal of thermal, chemical and radioactive waste in the atmosphere, rivers etc.
- **In the space field:** calculation of drag and lift.

- **In cooling-heating ventilation of buildings:** improvement of living conditions and physiology, prediction of the air and blood flow through the kidneys and arteries.

1.3   Mathematical Structure of Computational Codes

The mathematical formulation structure of computational codes follows the flow chart below.

```
┌──────────────────┐   ┌──────────────┐      ┌──────────────┐
│ Conservation Laws│   │ Transfer Laws│      │  Source Laws │
└──────────────────┘   └──────────────┘      └──────────────┘
              │                │                     │
              ▼                ▼                     ▼
                                          ┌───────────────────────┐
                                          │ Differential Equations│
                                          └───────────────────────┘
                    ┌────────────────────────────┐
                    │ Finite Difference Equations │
                    └────────────────────────────┘
                                 │
                                 ▼
                    ┌────────────────────┐
                    │  Algorithm Solution│
                    └────────────────────┘
                                 │
                                 ▼
                    ┌────────────────────┐
                    │  Computer Program  │
                    └────────────────────┘
                                 │
                                 ▼
                    ┌────────────────────┐
                    │  Equations Solution│
                    └────────────────────┘
                                 │
                                 ▼
                        ┌─────────────┐
                        │ Predictions │
                        └─────────────┘
```

As shown in the flow chart, the computational codes are based on the laws of nature, conservation (momentum, mass and energy), transfer and sources. The laws of nature are transformed directly into finite differences, using the analysis of the control-volume method. Tending the number of the grid nodes into infinity, the approximation of the finite difference equations formed can replace that of the differential equations.
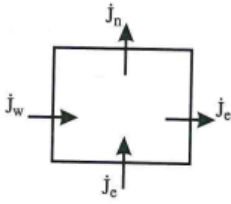
In order to enable the prediction through a Computer and resolve the finite difference equations, a resolution algorithm is necessary, which should be properly integrated in the computer program, take advantage of its speed and convey the physical reality.

CHAPTER 2

GOVERNING EQUATIONS OF

FLUID MECHANICS

2.1 Conservation Equations – Partial Differential Equations

In order to show how the laws of nature can be combined in a differential equation, the momentum transfer is taken, for example. According to Newton's second law for the constant flow state, the sum of the momentum (J_i) in the i direction must be equal to the net force in the same direction (S_i). The mathematical expression is:

$$\dot{J}_{i,w} - \dot{J}_{i,e} + \dot{J}_{i,s} - \dot{J}_{i,n} = - S_i$$

The mathematical expression includes (j) flows, which represent the momentum transfer by both transfer and diffusion (viscosity effect). They have as their cause the transfer laws (Newton's law of viscosity). According to the transfer law, in a Newtonian turbulent fluid, the total flow is equal to:

$$\dot{J}_x = \varrho UU - \mu t \left(\frac{\partial U}{\partial x}\right) + \quad \text{additional terms}$$

(2.1.1)

In the (2.1.1) relation there are also additional terms (or sources) that contribute to the momentum transfer, which arise from the "source laws" and describe the contribution due to pressure, buoyancy forces etc. Source law:

$$S_x = \frac{\partial p}{\partial x} + S'_x$$

(2.1.2)

Replacing the laws of transfer and sources in the mathematical expression of Newton's second law, we obtain the differential equation for momentum in the x- direction:

$$\frac{\partial}{\partial x}(\varrho UU) + \frac{\partial}{\partial y}(pUV) - \frac{\partial}{\partial x}\left(\mu_t \frac{\partial U}{\partial x}\right) - \frac{\partial}{\partial y}\left(\mu_t \frac{\partial U}{\partial y}\right) = \frac{\partial p}{\partial x} + S'_x$$

(2.1.3)

The differential equations for momentum in the other directions, as well as for other conserved properties, are obtained in the same way.

The beauty of transfer equations in the conservation of various properties (excluding the mass), is that they can be expressed by a general formula, which for two-dimensional steady-state problems (constant flow), for cylindrical coordinates is as follows:

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(\varrho r U\Phi) + \frac{\partial}{\partial r}(\varrho r V\Phi) - \frac{\partial}{\partial x}\left(r\Gamma \frac{\partial \Phi}{\partial x}\right) - \frac{\partial}{\partial r}\left(r\Gamma \frac{\partial \Phi}{\partial r}\right)\right] - S_\Phi = 0$$

(2.1.4)

$\Phi = U, V, k, \varepsilon, T, m_j$, etc., $\Gamma = \mu_t, \Gamma_k, \Gamma_\varepsilon, \Gamma_\tau$, etc. $S_\varphi$represents the sources in relation to the transfer of the Φ variable. The continuity equation, or, in other words, conservation of mass, has a special expression and will be discussed later, after the production of the corrective pressure equation. For r = 1 and $\partial r = \partial y$ we go to a two-dimensional flat flow.
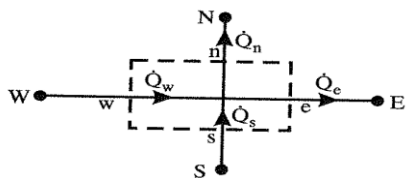
2.2   Finite Difference Equations

For the production of finite difference equations from differential equations the appropriate grid and the storage positions of the variables must be developed. The grid used in the plane r-x is a regular rectangle, with random distances of the nodes $\delta x_{PW} \neq \delta x_{EP}$ and is represented by the continuous lines:
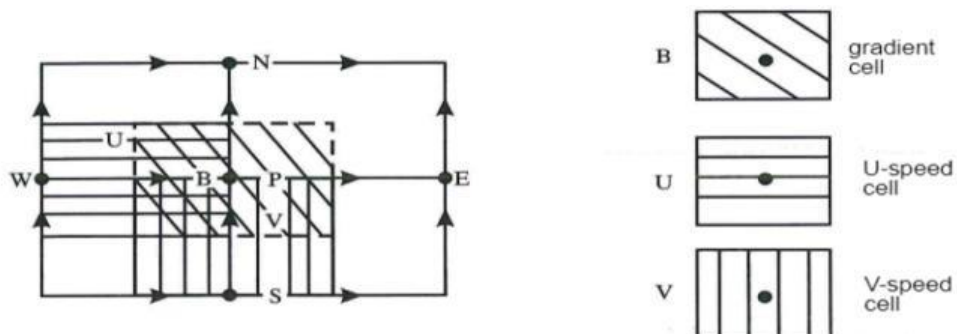
Typical clusters of U, V and gradient cells (or control volumes) are represented by the dashed lines. Each cell surrounds the position of the relevant variable. The variables are stored in different grid positions. The pressure and the gradients are stored at the grid nodes, while the speeds are

stored at the limits of the gradient cells. This storage system, known as shifted "staggered" grid, has the advantage that the variables U, V, P are stored in such a way that the pressure slopes leading the U and V speeds are easy to be estimated and in addition, the speeds are stored where needed to calculate the transferred flow. In $(\bullet)$ positions the gradient variables P, k, ε, T, m_j are stored, in $(\rightarrow)$ positions the U speed and in $(\uparrow)$ positions the V speed.
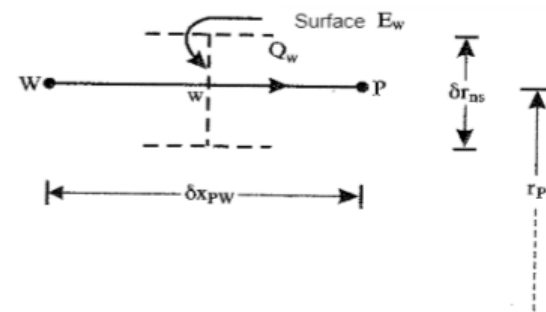
The approach using the control volume (or cell) method is similar to the holistic method, but more related to physics. The value of some Φ property reported at the node point refers to the average value of the control volume. The conservation law for the transfer of some expansive Φ property (mass, momentum, energy etc.), can be defined: [(the change of Φ in the cell) = (the net introduction rate of Φ into the cell by diffusion) + (the production rate of Φ within the cell)]. And mathematically:

$$\frac{\Delta(\varrho\Phi)}{\Delta t}\,dV + \dot{Q}_w - \dot{Q}_e + \dot{Q}_s - \dot{Q}_n^{''} + \int_V S_\Phi\,dV = 0$$

w, e, s, n, represent the limits of the cells. Q_i the total flow due to transfer and diffusion, as a sum around the limits of the cells, giving a natural perception and emphasizing on the conservation. The term S_Φ represents the production in the volume unit in the cell and Φ = U, V, T, m_j, k, ε (Φ=1 for the mass). $\Delta(\rho\Phi)/\Delta t = 0$, for steady-state flow (constant flow).

Considering, for the sake of convenience, a one-dimensional transfer across the cell limits, the "exact" method of Spalding (1972) for solving the western limit of the cell gives:

$$\dot{Q}_w \approx \varrho_w U_w E_w \left[f_w \Phi_W + (1 - f_w)\,\Phi_P \right]$$

$$f_w \equiv \frac{\exp(P_{ew})}{\exp(P_{ew}) - 1},\qquad (.\ \text{Peclet Number}\ )\ P_{ew} \equiv \frac{p_w U_w \delta x_{PW}}{\Gamma_w}$$

$$A_w = r_P \delta r_{ns},\qquad \varrho_w = \frac{\varrho_W + \varrho_P}{2},\qquad \Gamma_w = \frac{\Gamma_W + \Gamma_P}{2}$$

Q_w is the transferable gradient Φ, in the sense of the average value, obtained through the balancing coefficient (which depends on the local Peclet

number). If the density ρand the diffusion coefficient Γ are not stored in uniform grids, balanced values are used to confirm the flow continuity.

In order to avoid "computationally accurate" exponential relations, a method that uses the "piece-wise" (Patankar, 1980) linear approach is adopted, to calculate the "exact" $\dot{Q}_w$ P_{ew}relation (with a small loss of accuracy). According to this method, a "central" difference method for a low $|P_{ew}|$ (Peclet) number and an "upwind" difference method (asymptotic of the upwind relation) for a large $|P_{ew}|$ number are used. This is why the method is called mixed ("hybrid") method:

$$\frac{\dot{Q}_w}{\varrho_w U_w E_w} \equiv \begin{cases} \frac{1}{2}\left[(1 + 2P_{ew}^{-1})\,\Phi_W + (1 - 2P_{ew}^{-1})\,\Phi_P\right] & \gamma\iota\alpha \; -2 < P_{ew} < 2 \\ \Phi_W & \gamma\iota\alpha \; P_{ew} \geq 2 \\ \Phi_P & \gamma\iota\alpha \; P_{cw} \leq -2 \end{cases}$$

$\dot{Q}_e$, $\dot{Q}_n$and$\dot{Q}_s$arise in a similar way.

2.3   Source Term

The total production in the control volume cannot be accurately expressed, without knowing the exact expression of the source term S_Φ. However, we can give it a linear form:

$$-\int_V S_\Phi \, dV = b\Phi_P + c$$

b and c arise during the integration and linearization of the source term S_Φ, and are generally functions of Φ. This approach offers advantages and ease of use in the computational program, in various flow situations.

2.4   Final Finite Difference Equation

For the case of steady-state flow by replacing the flow and production expressions in the conservation law, we produce with the help of the continuity the **final finite difference equation:**

$$(\alpha_p - b)\Phi_p = \Sigma_i \alpha_i \Phi_i + c$$

α_n, α_s, α_e,and α_w combine diffusion transfer coefficients.

$\alpha_P = \Sigma_i\alpha_i$, $\Sigma_i = $ sum of the neighboring N, S, E, W.

$$\alpha_w = p_w U_w E_w f_w, \qquad\qquad \alpha_N = p_n U_n E_n f_n$$

$$\alpha_S = p_s U_s E_s f_s, \qquad\qquad \alpha_E = p_e U_e E_e f_e$$

With the help of the continuity, a_p expresses the sum of the combined flow coefficients at the nodes N, S, E, W. When b=c=0, Φ_P represents the average value of the sum of the neighboring nodes.

2.5   Introduction of Limit Conditions

The general finite difference equation is not applicable at the limits of the calculation area. Special treatment is required in the cells next to the limits, so that there is an advantage in both the conservation law and the flow calculation. The following figure shows a typical cell whose western limit coincides with a wall.
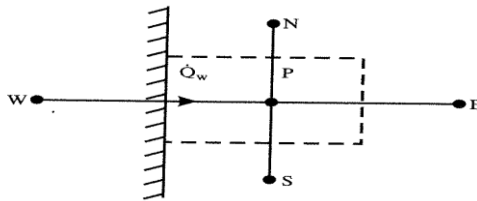
According to this arrangement, there is a connection between Φ_P and Φ_w in the general finite difference equation. The connection between Φ_P and Φ_w is interrupted, by setting the coefficient $\alpha_w = 0$. Also, we need to intervene in the flow $\dot{Q}_w$. The flows $\dot{Q}_e$, $\dot{Q}_n$ and $\dot{Q}_s$ according to the figure remain unaffected. There are various ways of intervention in $\dot{Q}_w$. Here, the treatment of the "False" (wrong) source is adopted by defining the constants b and c (easy in programming).

If we want to enter the limit flow Q_O, we set:

$$\alpha_w = 0, \quad b = 0, \qquad\qquad c = \dot{Q}_o$$

If we want to enter the limit value Φ_O, there must be:

$$\dot{Q}_w = \alpha'_w (\Phi_o - \Phi_P)$$

and we set:

$$\alpha_w = 0, \quad b = -\alpha'_w, \quad c = \alpha'_w \Phi_o$$

If the relative limit in the calculation area is a wall, α'_w must be estimated based on the wall functions, or by another model describing the flow near the wall.

Many times, Φ needs to have a constant value within the calculation area (secondary infusion set within the calculation area). The "False" source treatment is a very useful tool for these cases. We set:

$$b = -\gamma, \quad c = \gamma \Phi_{fix}$$

where γ (= large number), e.g. 10^{30} and $\Phi_{fix} = n$ the desired constant value within the calculation area.

2.6   Momentum Finite Difference Equation

The previous finite difference equation that has arisen was based on gradient variables. The momentum finite difference equations are produced similarly with the only exception of the control value shift, because the speeds are also shifted. The U-equation mentioned in the cell is:

$$(\alpha_P - b)\, U_P = \Sigma_i \alpha_i U_i + E_{ew}\,(P_W - P_P) + c$$
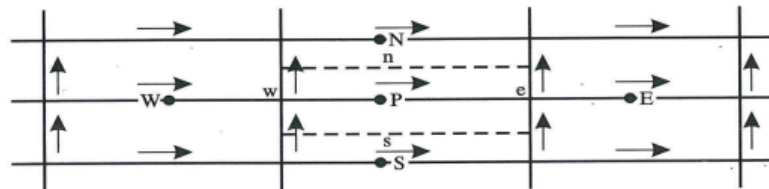
$$\alpha_w = \varrho_w U_w E_{ew} f_w, \quad E_{ew} = r_P \delta r_{ns}, \quad f_w = f_{Hybrid - Difference}\,(Re_w)$$

$$Re_w = \frac{\varrho_w U_w \delta x_{PW}}{\mu_w}, \quad \varrho_w U_w = \frac{1}{2}\,(\varrho_w U_{W'} + \varrho_P U_P)$$

We will refer to the pressures in the chapter of the corrective pressure equation. The speeds at the limits of the cell, the densities etc., are calculated by interpolation, in order to satisfy the continuity for the total flow.

CHAPTER 3

RESOLUTION OF FINITE DIFFERENCE EQUATIONS

3.1   Resolution of Finite Difference Equations

This chapter describes the general "Line-By-Line" (ILBL), iterative process for resolving all finite difference equations and a special (SIMPLE) algorithm for hydrodynamic equations.

3.2   LBL Resolution of Finite Difference Equations using the TDMA Algorithm
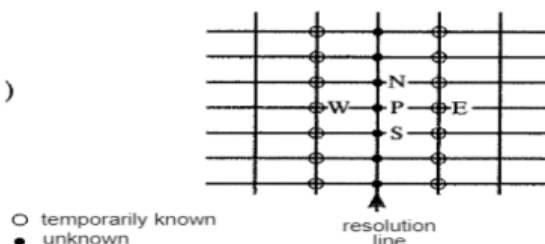
In general, the Line-By-Line process is an iterative method, with initial assumption of the solution field values and line-by-line solution improvement.

When solving equations for the points on the same line (e.g. N-S line), the values of the neighboring lines are considered temporarily known. The equation is then transformed for each point on the N-S line into such a form that only three values (Φ_P, Φ_N and Φ_S) are unknown.

$$\alpha'_P \Phi_P = \alpha_N \Phi_N + \alpha_S \Phi_S + c'$$

$$c' = \alpha_W \Phi_W + \alpha_E \Phi_E + c \ (\text{known})$$

$$\alpha'_P = \alpha_P - b$$

○ temporarily known
● unknown

resolution line

The system of equations for all the points on the N-S line takes a very simple form and the table of the non-zero coefficients is tri-diagonal. Φ_1 and Φ_{i+1} are generally known in applications.

$$-\beta_2\Phi_1+D_2\Phi_2-\alpha_2\Phi_3 \qquad\qquad = c_2'$$

$$-\beta_3\Phi_2+D_3\Phi_3-\alpha_3\Phi_4 \qquad\qquad = c_3'$$

$$-\beta_j\Phi_{j-1}+D_j\Phi_j-\alpha_j\Phi_{j+1} \qquad\qquad = c_j'$$

$$-\beta_i\Phi_{i-1}+D_i\Phi_i-\alpha_i\Phi_{i+1} \qquad\qquad = c_i'$$

$$D \equiv \alpha'_P, \qquad \alpha \equiv \alpha_N, \qquad \beta \equiv \alpha_S, \qquad \Phi_1,\ \Phi_{i+1} = \text{known.}$$

Equations of this type are easily resolved with the TDMA algorithm (Tri-Diagonal Matrix Algorithm), from the points j=2, to j=i, on the N-S line.

For the needs of the TDMA algorithm, the system of equations with algebraic manipulation is converted into general iterative relations for Φ_j and the coefficients A_j and C''_j. Properly handling the j-th equation we obtain:

$$\Phi_j = Q_j\Phi_{j+1} + R_j\Phi_{j-1} + Z_j$$

where:

$$Q_j = \frac{\alpha_j}{D_j}, \quad R_j = \frac{\beta_j}{D_j}, \qquad Z_j = \frac{C'_j}{D_j}$$

The equations take the form:

$$\Phi_2 = Q_2\Phi_3 + R_2\Phi_1 + Z_2 \qquad\qquad \text{(i)}$$

$$\Phi_3 = Q_3\Phi_4 + R_3\Phi_2 + Z_3 \qquad\qquad \text{(ii)}$$

$$\Phi_4 = Q_4\Phi_5 + R_4\Phi_3 + Z_4 \qquad\qquad \text{(iii)}$$

$$\Phi_n = Q_n\Phi_{n+1} + R_n\Phi_{n-1} + Z_n$$

Φ_1 is known. Deleting Φ_2 from (ii) and Φ_3 from (iii) and so on, gives a general formula for Φ_j:

$$\Phi_j = A_j\Phi_{j+1} + C''_j$$

where:

$$A_j = \frac{\alpha_j}{D_j - \beta_j A_{j-1}}, \qquad C''_j = \frac{\beta_j C''_{j-1} + C'_j}{D_j - \beta_j A_{j-1}}$$

Note: $A_1 = 0, \qquad C''_1 = \Phi_1.$

By applying the TDMA algorithm to the N-S line, A_j and C''_j from j=2 to j=n are calculated from the iterative relations. From the general iterative relation we take Φ_j, starting with Φ_n and ending with Φ_2 (Φ_1, Φ_{n+1} known). By applying the TDMA algorithm to the whole field, the calculation starts e.g. from

the extreme N-S line. Then, the calculation is repeated along the next neighboring N-S lines, using the recently calculated Φ_j values by integrating them into C´. The whole grid is scanned (scan = movement from one grid line to another) and we may use multiple scans, in order to achieve the desired solution. Divergences from the crossings direction (crossing = movement along a defined grid line) and the scans are possible.

3.3   SIMPLE Algorithm

The unknown variables to be solved are the main hydrodynamic variables U, V, P and the additional gradient variables k, ε, T, $m_j$etc. Each unknown variable requires resolving an equation. The gradient variables satisfy this requirement. And the speeds U, V, also satisfy this requirement (with the momentum equations). The pressure, however, has no equation. There is an additional equation, the continuity equation, but the pressure term is not in it. Therefore, a special methodology is required, in order to obtain the pressure P. The methodology used here is to resolve the momentum equations initially, having estimated a pressure field. After obtaining the estimated speeds U, V and, finally, the pressure field corrections, having in agreement the speed field with the continuity equation. This resolution process is known as SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm (Patankar and Spalding, 1972).

Initially, the field of the pressure P is assumed and the momentum equations are solved by the LBL method and corresponding speeds U*, V* are produced. The incorrect values of P*, U* and V* require correction P´, U´, and V´:

$$P = P^* + P', \quad U = U^* + U', \quad V = V^* + V', \quad G = G^* + G'$$

The shifted "staggered" grid system provides an advantage in the corrections of the speeds, or the flow (G´), with the expression of $G´_w$, $G´_e$, $G´_n$, $G´_s$ as coefficients of time slopes t´, of P´. We use a linearized flow relation to obtain $G_w´$ in terms of P´:

$$G'_w = \frac{f'_w(P'_P - P'_w)}{\delta x_{PW}}, \quad t'_w = -\varrho^* \delta x_{PW} \frac{\partial U_w^*}{\partial (P_P^* - P_W^*)},$$

$$G'_w = \varrho^* \frac{\partial U^*_w}{\partial(P^*_W - P^*_P)}(P'_W - P'_P)$$
and:
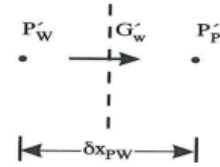
Using the momentum equations in terms of U*, V* and P*, the expression for G´w is finally transformed into a simple formula, with the coefficient D_w in terms of α_P and b:

$$G'_W = \varrho^* D_w (P'_W - P'_P), \quad D_w = \frac{\partial U^*_w}{\partial(P^*_W - P^*_P)} = \frac{E_{ew}}{(\alpha_P - b)}$$

For uncompressed flow G´ = ρ*U´. In the compressed flow we should be careful when calculating the densities. If the speeds are corrected, the mass conservation equation will be satisfied and the mass source S_m is equal to zero. Of course, at the originally assumed speeds U* and V* and therefore G* the continuity is not generally satisfied and there is a net source mass. For a typical control volume we have:

$$G_e E_e - G_w E_w + G_n E_n - G_s E_s = S_m \delta V$$
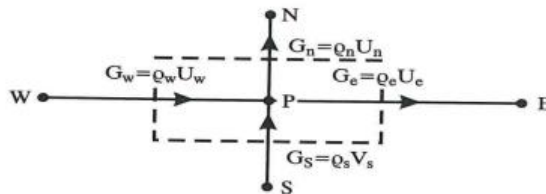
The purpose here is to correct the speeds and the pressures in such a way that the source mass is eliminated. Replacing $G_w = G^* + G'$ and so on, in the mass conservation equation, a Poisson equation for the corrective P´ arises.

$$(\alpha_P - b)P'_P = \Sigma_i \alpha_i P'_i + M_P + c$$

where:

$$\alpha_P = \Sigma_i \alpha_i, \qquad \alpha_w = p_w D_w E_w \text{ and so on.}$$
$$M_P = G^*_w E_w - G^*_e E_e + G^*_s E_s - G^*_n E_n \equiv \text{ residual mass source in}$$

relation to the assumed G* flows.

The resolution of the corrective pressure P´ with the LBL method completes the process of obtaining the corrections U´, V´ and P´ required forU*, V* and P*. At the limits of the solution fields, if there is a speed vertical

to the limit, no pressure corrections are needed. E.g. at the western limit the coefficient D_w must be zero and this is achieved by setting $\alpha_w = 0$ in the pressure equation:

$$\mathbf{U'_w = U_{limit} - U^*_{limit} = D_w \, (P'_w - P'_P) = 0}$$

If the limit pressure is given, e.g. $P_w = P_{LIMIT}$, the corrective pressure P'_w is equal to zero. Then U_w and D_w are obtained in a similar way, or from the momentum equation (e.g. by the linearization of the Bernoulli equation

$$\left(P = P_{LIMIT} + \frac{1}{2} \varrho U_w^2 \right):$$

$$P_W = P_{limit}, \qquad P'_W = 0$$
$$U_w = \alpha P_P + \beta, \qquad U'_w = \alpha P'_P$$

The various components of the SIMPLE algorithm previously mentioned are now combined with the resolution of the equations of the non-hydrodynamic variables, in order to achieve an overall combined resolution process. The field of all the variables (U, V, P, T, k, ε, and so on) is assumed. The coefficients of the momentum equations are calculated and the improved values U*, V* are obtained by the LBL method, using prevailing pressures:

$$(\alpha_P - b) U_P^* = \Sigma_i U_i^* + E_{ew} \, (P_W^* - P_P^*) + c$$

More than one scan may take place, but without the timeout rates of the coefficients. At this point the momentum equations are satisfied, but not the continuity equation. Then, the coefficients of the corrective equation P′ are calculated and this equation is resolved by the LBL method. Usually, more than one scan is required for the corrective pressure equation without the timeout rates of the coefficients. Next, the corrective speeds U′ and V′are calculated and P, U and V are obtained from the relations:

$$P = P^* + P', \quad U = U^* + U', \quad V = V^* + V'$$

At this point the continuity is exactly satisfied but not the momentum equations. In the next step the coefficients of the non-hydrodynamic equations are calculated and the relative gradients Φ are resolved by the LBL method. The number of scans required in the same time step depends on the nature of the problem.

Finally, a convergence test is performed, which if not successful, then the recently obtained fields of the variables are considered assumed and the process is repeated until final convergence.

Then, the SIMPLE algorithm methodology for the case of the two-dimensional constant flow for Cartesian coordinates is briefly described.

The continuity equation is:

$$\frac{\partial \varrho}{\partial t} + \frac{\partial \varrho u}{\partial x} + \frac{\partial \varrho v}{\partial y} + \frac{\partial \varrho \omega}{\partial z} = 0$$

For steady-state (constant) two-dimensional flow we have:

$$\frac{\partial \varrho}{\partial t} = 0 \,, \qquad\qquad \frac{\partial \varrho \omega}{\partial z} = 0$$

The continuity equation becomes:

$$\frac{\partial \varrho u}{\partial x} + \frac{\partial \varrho v}{\partial y} = 0 \qquad\qquad \frac{\Delta \varrho u}{\Delta x} + \frac{\Delta \varrho v}{\Delta y} = 0$$

With integration in the control volume:

$$\int_v \left(\frac{\partial \varrho u}{\partial x} + \frac{\partial \varrho v}{\partial y} \right) dV = 0 \;\Rightarrow\; (\Delta \varrho u)\, \Delta y \Delta z + (\Delta \varrho v)\, \Delta x \Delta z = 0$$

$$\left((\varrho u)_e - (\varrho u)_w \right) \Delta y \Delta z + \left((\varrho v)_n - (\varrho v)_s \right) \Delta x \Delta z = 0$$

Using an uncorrected pressure field p*, on the one hand, the momentum equations with the calculation of u* and v* are satisfied, but not the continuity equation. We have:

$$p = p^* + p'$$ P = actual, P^* = assumed, P' = corrective pressure.

The speeds can be corrected through the relations:

$$u = u^* + u', \qquad v = v^* + v'$$

The corrective speed equations are produced by subtracting the actual solution from the originally assumed one:

$$\alpha_e u'_e = \Sigma\, \alpha_i u'_i + A_e \left(p'_p - p'_e \right)$$

If the term $\Sigma \alpha_i u'_i$ is set equal to zero, it indirectly gives the equation the correct solution (the equation is valid in case of convergence).

$$u'_e = \frac{A_e}{\alpha_e} \left(p'_p - p'_e \right) = d_e \left(p'_p - p'_e \right) \qquad\qquad \text{where} \qquad d_e = \frac{A_e}{\alpha_e}$$

The corrective speed formula:

$$u'_e = d_e \left(p'_P - p'_E \right) A_e$$

can also be written as:

$$u_e = u_e^* + d_e \left(p'_P - p'_E \right)$$

Replacing the actual speed in the continuity equation:

$$\left[\varrho_e \left(u_e^* + u'_e \right) - \varrho_w \left(u_w^* + u'_w \right) \right] \Delta y \Delta z +$$

$$+ \left[\varrho_n \left(u_n^* + u'_n \right) - \varrho_s \left(u_s^* + u'_s \right) \right] \Delta x \Delta z = 0$$

Replacing the corrective speeds in terms of corrective pressures:

$$0 = \Delta y \Delta z \left\{ \varrho_e \left[u_e^* + d_e \, (p'_P - p'_e) \right] - \varrho_w \left[u_w^* + d_w \, (p'_w - p'_P) \right] \right\} +$$

$$+ \Delta x \Delta z \left\{ \varrho_n \left[u_n^* + d_n \, (p'_P - p'_n) \right] - \varrho_s \left[u_s^* + d_s \, (p'_s - p'_P) \right] \right\}$$

We come to the final expression for the corrective pressure equation:

$$\alpha_P p'_P = \alpha_E p'_e + \alpha_W p'_w + \alpha_S p'_s + \alpha_N p'_n + b$$

$$\alpha_E = \varrho_e d_e \, \Delta y \Delta z$$

$$\alpha_W = \varrho_w d_w \, \Delta y \Delta z$$

$$\alpha_S = \varrho_s d_s \, \Delta x \Delta z$$

$$\alpha_N = \varrho_n d_n \, \Delta y \Delta z$$

$$\alpha_P = \alpha_E + \alpha_W + \alpha_S + \alpha_N$$

$$b = \left(-\varrho_e u_e^* + \varrho_w u_w^* \right) \Delta y \Delta z + \left(-\varrho_n u_n^* + \varrho_s u_s^* \right) \Delta x \Delta z$$

And in case there is a time limit in the continuity equation:

$$\frac{(\varrho_P - \varrho_P^O) \, \Delta x \Delta y}{\Delta t}$$

The term b becomes:

$$b = \frac{(\varrho_P - \varrho_P^O) \, \Delta x \Delta y}{\Delta t} + \left(\varrho_w u_w^* - \varrho_e u_e^* \right) \Delta y \Delta z + \left(\varrho_s u_s^* - \varrho_n u_n^* \right) \Delta x \Delta z$$

3.4 Method Convergence

All equations satisfy conditions for table convergence (Scarborought, 1930):

$$\left| \alpha_P - b \right| \le \Sigma_i \left| \alpha_i \right|, \qquad \text{with } b \le 0$$

During the resolution process, the convergence is determined at the end of each repetition, based on the "residual source" criterion, which compares the "residual values" of each finite difference equation with each reference value $R_{\Phi \text{ reference}}$, ($R_{\Phi, \text{reference}} = $ a very small value).

By using a suitable under-relaxation method for an iterative process, convergence can be improved and divergence is often avoided. The equations that are solved are non-linear and the under-relaxation is necessary:

$$\Phi_P^{NEW} = f\,\Phi_P + (1 - f)\,\Phi_P^{OLD}$$

f = under-relaxation coefficient,

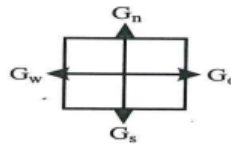Φ_P^{NEW} = current variable with under-relaxation,

Φ_P = current variable,

$\bar{\Phi}_P^{OLD}$ = previous repetition variable.

If the mass flows do not satisfy the continuity, a situation may arise where all α_j are equal to zero. The finite difference equations take the form:

$$\alpha_P = \Sigma_i \alpha_i$$

The solution is to add an "incorrect" (false) source, by the linearized source method. The final finite difference equation then has as additional constants b_j and c_j. The linearized source S_f is :

$$S_f = \left| \dot{m}_{net} \right| \left(\Phi_P^{OLD} - \Phi_P \right) = b_f \Phi_p + c_f$$

where: $m_{net} = \Sigma_i \dot{m}_i$, with $\dot{m}_w = G_w E_w$ and so on. The finite difference equation formula to be solved becomes:

$$(\alpha_P - b - b_f)\,\Phi_p = \Sigma_i \alpha_i \Phi_i + c + c_f$$

This addition helps to stabilize the resolution process, without affecting the final solution.

3.5   Method Accuracy

The accuracy of the solution process, in general, is a function of the convergence and the number of the grid nodes used. For each flow configuration, a grid-independent solution is sought, increasing the number of the grid lines, until no further change in the final solution is observed.

The main source of an incorrect prediction is the "incorrect" (false) diffusion that occurs, when the Peclet number is large and the flow is lateral to the grid (in the finite difference method, Φ_P is calculated as the average value of the neighboring Φ_I). As a solution, the grid could be placed parallel to the flow lines or all the Peclet numbers could be reduced.

CHAPTER 4

GOVERNING EQUATIONS FOR TURBULENT FLOW

4.1 Turbulent Model of Two Equations

This chapter presents the integration into the process of resolving a turbulent model, to obtain a solution of the equations through average time values of the average flow, with the help of two turbulent quantities k and ε, which are obtained from their transfer equations. According to the view of the Reynolds average time values, the instantaneous values of the speeds and the gradient variables are analyzed in the average values and their fluctuations:

$$\hat{U}_i = U_i + u_i, \ \hat{\Phi} = \Phi + \phi'$$

For steady-state flows and using Cartesian coordinates the equations of average time values for the continuity, momentum and gradient transfer take the expression:

$$\frac{\partial}{\partial x_i}(\varrho U_i) = 0$$

$$\frac{\partial}{\partial x_j}(\varrho U_j U_i) = \frac{\partial}{\partial x_j}\left(-\varrho \overline{u_i u_j}\right) - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] + S_{ui}$$

$$\frac{\partial}{\partial x_j}(\varrho U_j \Phi) = \frac{\partial}{\partial x_j}\left(-\varrho \overline{u_j \phi'}\right) + \frac{\partial}{\partial x_j}\left(\frac{\mu}{\sigma_\Phi}\frac{\partial \Phi}{\partial x_j}\right) + S_\Phi$$

μ= laminar viscosity, σ$_\Phi$= laminar Prandtl/Schmidt number.
Unfortunately, these equations contain unknown Reynolds stresses $\varrho \overline{u_i u_j}$ and gradient flows $\varrho \overline{u_j \phi'}$. These turbulent diffusion flows play an important role in determining the flow behavior, as they represent the effects at the micro-scale level.

The adopted method for obtaining a closed equation system is a model of "active" viscosity, in which the unknown turbulent diffusion flows are expressed in terms based on the "transfer slope hypothesis" (Hinge, 1959), in

which the flows are assumed to be proportional to the slopes of the average flow properties. The constants of the proportion are μ_t or $\mu_t/\sigma_{\varphi,t}$.

$$- \varrho \, \overline{u_i u_j} = \mu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \varrho \, \overline{u_j \phi'} = \frac{\mu_t}{\sigma_{\Phi,t}} \frac{\partial \Phi}{\partial x_j}$$

μ_t = turbulent viscosity,

$\sigma_{\Phi,t}$ = active Prandtl/Schmidt number.

$\sigma_{\Phi,t}$ is often considered to be known, while from the dimensional analysis it is concluded that μ_t is a function of the turbulent kinetic energy k and its scattering rate ε. k and ε are obtained from their transfer equations, which is why it is called turbulent model of k-ε two equations:

$$\mu_t = C_\mu \, \varrho \, \frac{k^2}{\varepsilon}$$

C_μ = friction coefficient, based on experimental measurements C_μ = 0,09.

4.2   Equations of Average Time Values in Turbulent Flow

The equations of average time values for the continuity, momentum and gradient transfer for two-dimensional, steady-state, turbulent axis-symmetric (or flat, r=1) flows are given:

Continuity:

$$\frac{\partial}{\partial x} (r \varrho U) + \frac{\partial}{\partial r} (r \varrho V) = 0$$

Momentum:

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\varrho UU) + \frac{\partial}{\partial r}(r\varrho VU)\right] = -\frac{\partial P}{\partial x} + \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\mu_{eff}\frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial U}{\partial r}\right)\right] + S_u$$

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\varrho UV) + \frac{\partial}{\partial r}(r\varrho VV)\right] = -\frac{\partial P}{\partial r} + \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\mu_{eff}\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial V}{\partial r}\right) - \mu_{eff}\frac{V}{r^2}\right] + S_V$$

$$S_u = \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial U}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial V}{\partial r}\right)$$

$$S_V = \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial U}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial V}{\partial r}\right) - \mu_{eff}\frac{V}{r^2}$$

$$\mu_{eff} = \mu + \mu_t, \qquad \Gamma_{eff} = \frac{\mu}{\sigma_\Phi} + \frac{\mu_t}{\sigma_{\Phi,t}}$$

Gradient Transfer:

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\varrho U\Phi) + \frac{\partial}{\partial r}(r\varrho V\Phi)\right] = \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\Gamma_{eff}\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\Gamma_{eff}\frac{\partial \Phi}{\partial r}\right)\right] + S_\Phi$$

Here μ_{eff} and Γ_{eff} are the active exchange coefficients, representing the sum of the laminar and the turbulent transfer. At high Reynolds numbers (full turbulent flow) the effects of the molecular transfer μ and Γ are negligible and are omitted in the equations. The expressions S_u, S_V omit the additional terms related to non-uniform viscosity, whose effect is beginning to become significant for significant changes of the fluid properties.

The necessary equations for the turbulent kinetic energy k and its scattering rate ε, which complete the equations of average time values for two-dimensional, steady-state axis-symmetric flow, are:

Turbulent energy k:

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\varrho Uk) + \frac{\partial}{\partial r}(r\varrho Vk)\right] = \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\frac{\mu_{eff}}{\sigma_k}\frac{\partial k}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\frac{\mu_{eff}}{\sigma_k}\frac{\partial k}{\partial r}\right)\right] + \underbrace{G - C_D\varrho\varepsilon}_{S_\varkappa}$$

Scattering rate of energy ε:

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(r\varrho U\varepsilon) + \frac{\partial}{\partial r}(r\varrho V\varepsilon)\right] = \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\frac{\mu_{eff}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\frac{\mu_{eff}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial r}\right)\right] + \underbrace{C_1\frac{\varepsilon}{k}G}_{S_\varepsilon} - C_2\varrho\frac{\varepsilon^2}{k}$$

where:

$$G = \mu_t \left\{ 2 \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial r}\right)^2 + \left(\frac{V}{r}\right)^2 \right] + \left(\frac{\partial U}{\partial r} + \frac{\partial V}{\partial x}\right)^2 \right\} + S_G$$

$$S_G = -\frac{2}{3} \mu_t \left[\frac{1}{r} \frac{\partial}{\partial r} (rV) + \frac{\partial U}{\partial x} \right]^2$$

The constants (Launder and Spalding, 1974):

C_μ	C_D	C_1	C_2	σ_k	σ_ε
0,09	1,0	1,44	1,92	1,0	1,3

The similarity of these equation to those of the average flow is

demonstrated by replacing $S_\varkappa = G - C_D \varrho \varepsilon$ and $S_\varepsilon = C_1 \frac{\varepsilon}{k} G - C_2 \varrho \frac{\varepsilon^2}{k}$.

G represents the production of k from the average flow, through the turbulent shear stress, and to be precise, ε is the scattering rate of the viscosity of k in heat by very small turbulent vortices. The coefficients C and the Prandtl numbers σ are generally empirical functions, but they are taken as constants for high Reynolds numbers. S_G has other production terms too, the effects of which are small, out of flows of non-uniform properties.

4.3   Limit Conditions

At the input of the computational area of the flow, the variables U, V, Φ, k, ε, can be determined by the good knowledge of the particular flow state (experimental measurements) or be estimated. E.g. the scattering rate ε can be estimated by the dimensional analysis, based on the fact that the turbulence is characterized by the energy of k and a scale of L length, which represents the size of the turbulent vortices $(\varepsilon \sim K^{3/2}/L)$.

At the output of the computational area (for large Reynolds numbers), the identification of the variables is not important. The usual practice is to set vertical slopes equal to zero and obtain the output speeds from the mass balance. Near the wall the local Reynolds becomes too small and the turbulent model is insufficient (it is designed for high Reynolds numbers). This, in combination with the abrupt change of the variables near the wall, makes the selection of the position of the grid nodes near the wall careful.

The average flow equations U, V, P´ and Φ of the laminar flow are converted for the turbulent flow, by replacing μ with μ_{eff}, Γ with Γ_{eff} and introducing some additional source terms, using source linearization. The

equations k and ε are not different from the other gradient transfer equations, considering S_k and S_ε as additional source terms, which are introduced by the source linearization method.

$$\int_V S_k\, dV = bk_P + c = -\frac{C_\mu C_D \varrho^2 k_P^* \delta V}{\mu_t} k_P + G\delta V$$

$$\int_V S_\varepsilon\, dV = b\varepsilon_P + c = \frac{C_2 \varrho \varepsilon_P^* \delta\, V}{k_P}\varepsilon_P + \frac{C_1 \varepsilon_P^* G\delta\, V}{k_P^*}$$

δV = cell volume, $k^*_p,\ \varepsilon^*_p$ = previous values.

$$G \cong 2\left[\left(\frac{U_e - U_w}{\delta x_{ew}}\right)^2 + \left(\frac{V_n - V_s}{\delta r_{ns}}\right) + \dots\right]$$

One point we should note about the treatment of the sources S_k and S_ε is that b must not be negative. This provides us with solution stability and confirms that the calculated k values never become negative.

Generally, near-wall conditions are considered to be one-dimensional Couette flow conditions. The limit layer is considered to have a constant shear stress ($\tau\tau_w$) and a constant heat flow ($\dot{q}'' = \dot{q}''_w$). These conditions require an impermeable wall with zero pressure slopes or negligible in the flow direction:

$$\frac{\tau_w}{dP/dx} \gg y$$

The momentum equation is then transformed into a special, simple, dimensionless form:

$$\tau = (\mu + \mu_t)\frac{dU}{dy} \qquad \text{or} \qquad \frac{\tau}{\tau_w} = \left(1 + \frac{\mu_t}{\mu}\right)\frac{dU^+}{dy^+}$$

In the area near the wall the local Reynolds number varies considerably and the adopted approach is the dependence of the local Reynolds number, y^+, which is based on the distance y from the wall and the friction speed U_t.

$$y^+ = \frac{U_\tau y}{\nu}, \qquad U_\tau = \sqrt{\frac{\tau_w}{\varrho}}$$

The area near the wall is divided into three sub-areas (Hinge, 1959). In the viscous substrate $0 < y^+ < 5$, where the effects of the viscosity predominate, in the inert substrate $30 < y^+ < 400$, where the flow is completely turbulent, but $\tau \sim \tau_\omega$ and in the transition layer $5 < y^+ < 30$, where the flow is not affected only by the viscosity but is not completely turbulent either. The

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flow separation is followed by the determination of the point, $y^+ = 11{,}63$ (where the linear distribution of the speed of the viscous substrate meets the logarithmic distribution of the inert substrate), below which the flow is assumed to be completely viscous, while the top is completely turbulent.

For $y^+ \leq 11{,}63, \quad \dfrac{\mu_t}{\mu} << 1, \quad \tau \sim \tau_w. \Rightarrow U^+ = y^+$

For $y^+ > 11{,}63, \dfrac{\mu_t}{\mu} >> 1, \tau \sim \tau_w, \ v_t \sim \varkappa y U_\tau \Rightarrow$

$$\Rightarrow U^+ = \frac{1}{\varkappa} \ln y^+ + \text{const} = \frac{1}{\varkappa} \ln(E y^+)$$

$U^+ = U/U_T$, $\kappa = 0{,}4187$ von Karman constant and E is an integration constant that depends on the size of the change in the shear stress across the limit layer and the roughness of the wall. The value $E = 9{,}8$ is valid for a smooth wall and a constant shear stress. Mass transfer effects across the limit layer and different pressure slopes are incorporated by modifying E.

Also, there is a great mechanical interest in predicting transfer characteristics to the walls. The same treatment, as for the momentum transfer, is applied to the heat transfer too. The corresponding dimensionless equation is:

Gradient transfer (e.g. $\Phi = T$):

$$\dot{q}'' = (\Gamma + \Gamma_t) \, c_p \frac{dT}{dy}, \qquad \frac{\dot{q}''}{\dot{q}''_w} = \left(\frac{\Gamma}{\mu} + \frac{\Gamma_t}{\mu} \right) \frac{dT^+}{dy^+}$$

Constant heat flow across the limit layer is assumed.

For $y^+ \leq 11{,}63, \quad \Gamma >> \Gamma_t, \quad \dot{q}'' \sim \dot{q}''_w \Rightarrow T^+ = \sigma_\Phi y^+$

For $y^+ > 11{,}63, \quad \Gamma < \Gamma_t, \quad \dot{q}'' \sim \dot{q}''_w \dfrac{\Gamma_t}{\varrho} = \dfrac{v_t}{\sigma_{\Phi,t}} \sim \dfrac{\varkappa y U_\tau}{\sigma_{\Phi,t}}, \Rightarrow$

$$\Rightarrow T^+ = \frac{\sigma_{\Phi,t}}{\varkappa} \ln y^+ + C_T(\sigma_\Phi) = \sigma_{\Phi,t} \left[U^+ + P\left(\frac{\sigma_\Phi}{\sigma_{\Phi,t}} \right) \right]$$

Where: $T^+ = \dfrac{\varrho U_\tau C_p (T_w - T)}{\dot{q}''_w}, \qquad \sigma_\Phi = \dfrac{C_p \mu}{\varkappa}, \qquad \sigma_{\Phi,t}$ turbulent Prandtl number

and:

$$P\left(\frac{\sigma}{\sigma_{\Phi,t}} \right) = 9{,}24 \left[\left(\frac{\sigma_\Phi}{\sigma_{\Phi,t}} \right)^{3/4} + 1 \right]$$

(Jayatillaka, 1966).

C_P = special heat under constant pressure, κ= conductivity coefficient.

As before, the Reynolds separation number is 11,63. For y^+< 11,63 it is assumed to take place with molecular activity. For y^+> 11,63 the transfer is assumed to be entirely due to turbulence. The heat flow parameter $T^+$is a logarithmic function of $y^+$and, also, of a term, which with the integration constant $C_T$is combined with the P-function (it is valid for impermeable smooth walls).

The wall treatment of the k and ε equations is again based on the view of a one-dimensional, constant shear stress Couette flow of a limit layer. The adopted approach is valid only in the inert substrate, where the flow is assumed to be completely turbulent, y^+> 30, but sufficiently close to the wall, so as the assumption of the constant shear stress to be valid (y^+< 400). In this area, the local rate of the turbulence production is balanced by the scattering rate of ε:

$$-\overline{uv}\,\frac{du}{dy} = \varepsilon \qquad \text{factor}$$

$$k = \left(\frac{\tau_l}{\varrho}\right) C_\mu^{1/2} \Rightarrow \tau_l = \varrho C_\mu^{1/2}\, k \qquad \text{shear stress in the inert substrate}$$

and: $$\varepsilon^{++} = \frac{C_\mu^{3/4}\, k^{3/2}}{\varkappa y}$$

The turbulent energy equation is transformed into a simple relation that describes the shear stressT_l(T_w) and the scattering rate ε, within the inert substrate.

The ε equation is transformed into a formula that represents a modification of σ_ε in this area:

$$C_1 = C_2 - \frac{\varkappa^2}{\sigma_\varepsilon C_\mu^{1/2}} \qquad \text{and:} \qquad \sigma_\varepsilon = \frac{\varkappa^2}{(C_2 - C_1)\, C_\mu^{1/2}}$$

One of the most important consequences of the equilibrium view is the final expression for the wall shear stress T_w, in terms of k and the turbulence constants.

$$\tau_w = \frac{\varrho C_\mu^{1/4}\, k^{1/2}\, \varkappa U}{\ln(Ey^+)}$$

$$\tau_w = \frac{U}{U^+}\varrho C_\mu^{1/4}\, k^{1/2}, \qquad U^+ = \frac{U\sqrt{\varrho\tau_l}}{\tau_w}, \qquad y^+ = \frac{y\sqrt{\tau_w/\varrho}}{v} \approx \frac{y\varrho\, C_m^{1/4}\, k^{1/2}}{\mu}\ ,$$

Also, the relation for ε with integration in the control volume (or cell), the adjacent to the wall (by extending the equilibrium in the k equation in the viscous substrate and the intermediate layer):

$$\int_V \varepsilon\, dV \approx \frac{C_\mu^{3/4}\, k^{3/2}\, U^+\, dV}{y}$$

with

$$U^+ = \begin{cases} y^+ & \text{for } y^+ \le 11{,}63 \\ \dfrac{1}{\varkappa} \ln(Ey^+) & \text{for } y^+ > 11{,}63 \end{cases}$$

Now, to integrate the wall limit conditions, a tangential speed U_P is assumed, at the usual momentum balance, for a node near the wall:
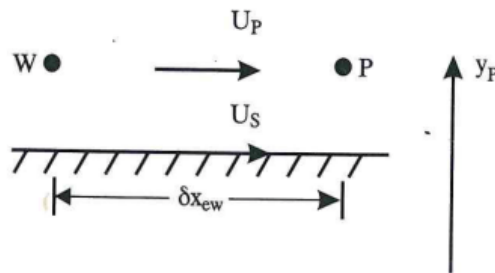
For this geometry $\alpha_s = 0$ is set. The incorporation of the correct expression of the shear force is introduced by the source method:

For the P node within the turbulent area $(y^+ > 11{,}63)$:

$$F_s = \tau_s \delta x_{ew} = \frac{\varrho C_\mu^{1/4} k_{pw}^{1/2}\, (U_p - U_s)\, \varkappa \delta x_{ew}}{\ln(Ey^+)}$$

where

$$k_{PW} = \frac{k_P + k_W}{2}, \qquad y^+ = \frac{\varrho C_\mu^{1/4}\, k_{PW}^{1/2}\, y_P}{\mu}$$

For the P node within the viscous substrate $(y^+ < 11{,}63)$:

$$F_S = \tau_S \delta x_{ew} = \frac{\mu\, (U_P - U_S)\, \delta x_{ew}}{y_P}$$

For the speeds vertical to the wall no special treatment is required.

The integration of the wall limit conditions for the gradient variables follows the same procedure, as in the case of the momentum. The expression α_s and the gradient flow Q_s are zeroed by the heat balance:

For P within the turbulent area $(y^+ > 11{,}63)$ we have:

$$\dot{Q}_S = \dot{q}'' \, \delta x_{ew} = \frac{\varrho c C_\mu^{1/4} \, k_P^{1/4} \, (T_P - T_S) \, \delta x_{ew}}{T^*}$$

where:
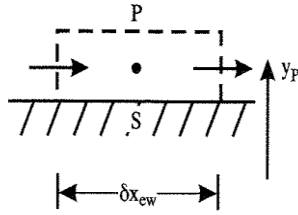
$$T^+ = \sigma_{\Phi,t} \left[U^+ + P \left(\frac{\sigma_\Phi}{\sigma_{\Phi,t}} \right) \right]$$

For P within the viscous substrate $(y^+ \leq 11{,}63)$:

$$\dot{Q}_S = \dot{q}'' \, \delta x_{ew} = \frac{\mu}{\sigma_\Phi} \frac{(T_P - T_S) \, \delta x_e}{y_P}$$

The integration of Q_S is again achieved by the source linearization method.

The integration of the turbulent kinetic energy k requires a special procedure. Using the balance for k we set $\alpha_S = 0$ on the wall. The production term G in the k-equation is transformed into a simpler form, depending on the shear stress $\tau_S \; (\sim \tau_w)$:

$$\int_V \mu_t \left(\frac{\partial U}{\partial r} + \frac{\partial V}{\partial x} \right)^2 dV \approx \frac{\tau_S \, (U_P - U_S) \, \delta V}{y_P}$$

Where $\tau_S, \; U_S$ are considered average values in the cell and:

$$\int_V C_D \varrho \varepsilon \, dV \approx \frac{C_D \varrho C_\mu^{3/4} \left(k_P^{*1/2} \, k_P - k_S^{3/2} \right) U^+ \delta V}{y_P}$$

where: $\quad U^+ = \frac{1}{\varkappa} \ln(E y^+) \quad$ for $\quad y^+ > 11{,}63$

and: $\quad U^+ = y^+ \quad\quad$ for $\quad y^+ \leq 11{,}63$

The whole source $S_k = G - C_D \varrho \varepsilon$ for the balance of k is incorporated into the code by the source method with the help of the coefficients b and c.

The wall flow, in contrast to k (which is zero), for ε reaches the maximum value (much higher than the free flow). This behavior makes it difficult to handle α_S in the balance of ε, in the cell near the wall. Here a set

value for ε_P is adopted (independent of y^+), based on equilibrium relations. Within the inert substrate (where the Re number is large) we have:

$$\varepsilon_P = \frac{C_\mu\, k_P^{3/2}}{L}, \qquad L = C_\mu^{1/4}\, \varkappa y \quad \text{in the wall area}$$

therefore:

$$\varepsilon_P = \frac{C_\mu^{3/4}\, k_P^{3/2}}{\varkappa y_P}$$

where: L is a turbulence scattering scale. The ε_P value is incorporated by the introduction method of the constants b and c (e.g. $b = 10^{30}$, $C = \varepsilon_P 10^{30}$).

4.4   Instability – Accuracy – Economy

In many complex flows, numerical instabilities may occur and then additional techniques are required, in order to achieve convergence. In simple flows there are three main causes of instability. The incorrect definition of the initial field may lead to instability. This can be eliminated by improving the initial field or by using under-relaxation coefficients. A second reason in the production of instability foci is the selection of inappropriate under-relaxation coefficients. To correct this, the under-relaxation coefficients are reset. A third reason is the incomplete resolution of the finite difference equations during the repetition. The corrective pressure equation P´is the most sensitive in this case, because in each repetition the initial field of P´ is zero. Increasing the number of application of the LBL method eliminates this instability.

Accuracy depends firstly on the degree to which the solution satisfies the finite difference equations. This degree is reflected in the stress of the residual sources. Secondly, on the degree to which the finite difference equations satisfy the partial differential equations. In general, smaller grid sizes are used to achieve a grid-independent solution. Thirdly, the conditions applied to the limits and the limit positions may affect the accuracy of the solution, as well. Improvement can be achieved by adjusting the conditions and the application positions. And fourthly, a fully convergent, grid-independgent solution, based on satisfactory limit conditions and application positions, depends on the adequacy of the turbulent model in how well the predictions reflect the reality compared to the experimental measurements.

Indeed, in complex flows the deficiency of the turbulent model may be the cause of various instabilities. It is important to distinguish between

computational errors and instability (which can be eliminated), and those errors due to physical modeling.

The requirements of economy, computational time and storage can be minimized with some techniques; first, by accurately identifying the initial fields (e.g. starting from a correct previous calculation, the computational time is significantly reduced); second, with optimal grid arrangement, concentration of the grid nodes in areas with abrupt slopes and reduction in areas, where the slopes are also useful for grid economy; third, by experimentally finding and testing the under-relaxation coefficients, improving the convergence factor; and fourth, based on a more realistic convergence criterion for complex flows, about 1% of the residual sources (in simple flows the criterion is maintained at 0,1%).

CHAPTER 5

COMPUTATIONAL PROGRAM STRUCTURE

### 5.1   Structure of the TEACH-T Computational Program

The TEACH-T educational program is a program for two-dimensional, steady-state, laminar or turbulent flows in Cartesian or cylindrical coordinates. In its normal form, it resolves for the variables U, V, P, k and ε (some extra variables are easily added).

The diagram below shows its overall structure:
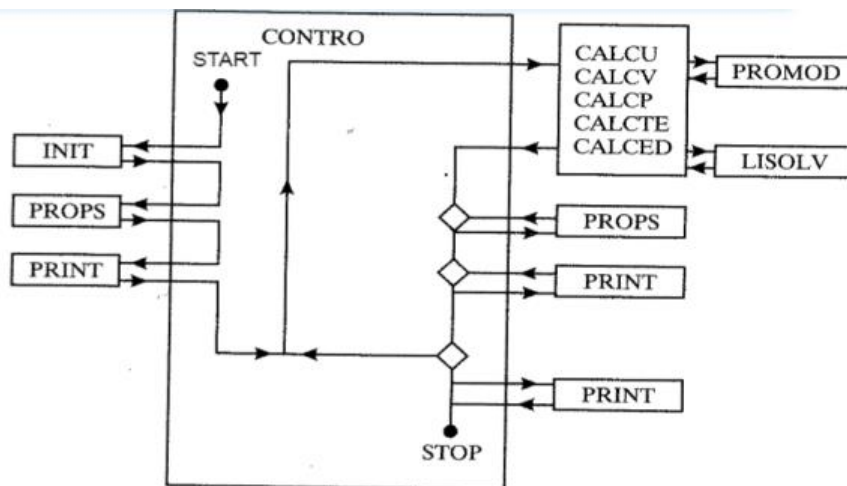
There are five general subroutines related to each particular variable to solve: CONTRO, INIT, PROPS, PROMOD, LISOLV and PRINT. In addition, there is the CALCΦ subroutine system. The program is generally controlled by the main subroutine CONTRO, which performs the initial and the final functions, while also controls the repetition. The subroutines CALCΦ perform the principal calculations of the finite difference equations for each Φ variable. Modifications of the sources and the limit conditions are made in PROMOD while PROPS calculates the fluid properties (viscosity, density, etc.). INIT performs the initial pre-processing, PRINT gives the output solutions and LISOLV performs the LBL repetition. The subroutines INIT, LISOLV, PRINT and CALCΦ are independent of the type of the problem. Appropriate modifications to each particular problem are required only in CONTRO, PROMOD and in rare cases in PROPS.

To demonstrate the TEACH-T computational code the STEP geometry (flow in abrupt expansion tube geometry) is obtained. The figure shows how the coordinates are determined.
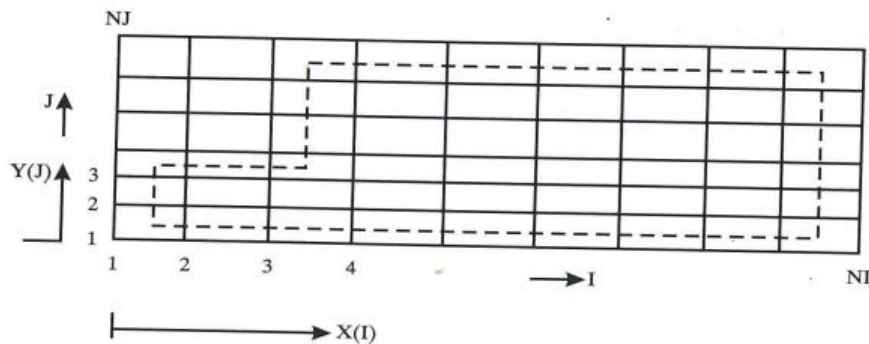
For cylindrical coordinates we have R(J) = Y(J), while for flat flows R(J) = 1. The calculation area is the rectangle limited by I = 2 to (NI − 1) and J = 2 to (NJ − 1). If desired, the calculation area can be changed. It should be noted that the limits of the flow area (dashed lines) always coincide with the limits of the main control volumes. The indicators (I, J) refer to the nodes whose coordinates are X(I) and Y(J). The figure shows the "staggered" grid storage system.
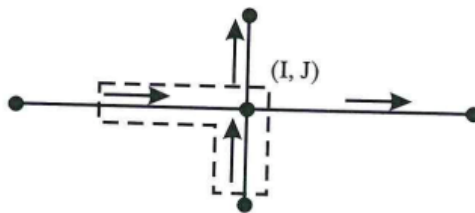
The gradient variables are stored at the main grid nodes (•), P → P(I, J), P´→ PP(I, J), k → TE(I, J), ε→ ED(I, J), μ_{eff}→ VIS(I,J), ρ→ DEN(I, J), while the relative speeds are shifted in the positions (→) and (↑), U → U(I, J) and V → V(I, J). The storage positions of the variables in the figure above are within the dashed lines. It should be noted that due to the storage way, the calculations for gradient variables start at the point (2,2), for the speed U at (3,2) and for the speed V at (2,3).

The finite difference equation resolution for some variable Φ (= U, V, P and so on), is obtained by setting INCALΦ = ".TRUE.". If we set INCALΦ = ".FALSE"., the corresponding equation is not resolved. Similarly, INPRO controls the calculation of the fluid properties. The indexing used to determine the application number of the LBL method, without time-outing the coefficients, for each variable Φ is NSWPΦ (number of scans). Often, it takes values from 1 to 6, depending on each Φ and the flow. The under-relaxation coefficients take values similarly to the factor URFΦ, which generally takes values between 0.5 and 1.0, depending on the particular Φ.

Monitoring repetitions, checking and extracting results give the user a better understanding of the success/failure of the computational process on a problem. For monitoring, the total number of the performed repetitions is stored as NITER and the absolute sum of the residual sources as RESORΦ. For the P′ equation the absolute mass source is stored as RESORM. RESOR are usually normalized. The repetition is checked so that the calculations are terminated for three reasons: the maximum residual source SORCE is too large after 20 repetitions (divergent solution); SORCE is smaller than the desired value SORMAX; NITER has reached the maximum allowed number MAXIT.

The P′ equation can be satisfied by different pressure fields. That is, the pressure is predetermined at the position (IPREF, JPREF) and is set to this value with all the other measured pressures at the other nodes related to that value. Thus, if this position is within the solution area, the pressure level is set.

In each repetition NITER, RESOR and the variables are printed in a specified position (IMON, JMON). The tables of the variables are printed before and after the continuation of the repetition. For printing within the continuation of the repetition, the variable INDPRI is used. The formula of the finite difference equations programmed is:

$$(\alpha_P - b - c_P)\, \Phi_P = \alpha_N \Phi_N + \alpha_S \Phi_S + \alpha_E \Phi_E + \alpha_W \Phi_W + c + c_P \Phi^{OLD}$$

Where:

$$\alpha_P = \alpha_N + \alpha_S + \alpha_E + \alpha_W, \qquad c_P = \max(0,\, M_P)$$

M_P is the net outflow from the control volume. c_P is used only as a trick that ensures stability, by the use of which the coefficients of Φ_P remain finite, in case of a net outflow.

The symbols of the FORTRAN language are carefully chosen, so that there is a direct relationship in their expression with their counterparts in the finite difference equations:

$$\alpha_P \rightarrow AP\,(I, J), \quad \alpha_N = AN\,(I, J), \quad \alpha_S = AS\,(I, J), \quad \alpha_W = AW\,(I, J)$$
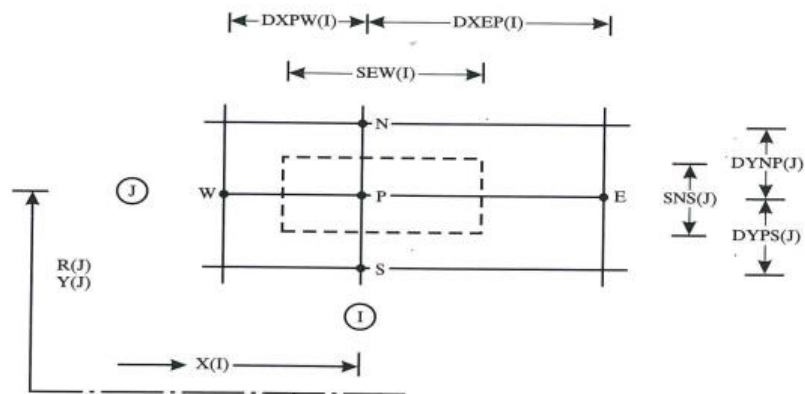
$$(c + C_P \Phi^{OLD}) \rightarrow SU\,(I, J), \qquad (b - c_P) \rightarrow SP\,(I, J)$$

or in the corrective speed formula:

$$D_u \rightarrow DU(I, J), \qquad D_V \rightarrow DV(I, J)$$

The following figure gives the cell (control volume) of the storage of the gradients (P, T, k, ε, and so on), with FORTRAN variables related to the dimensions/coordinates:


This figure, as well as the next two, is explanatory and the user will find in them great help in adopting TEACH-T for various problems. It should be noted that the gradient cell limits are located in the mid-range of the main grid nodes.

The following figure is explanatory for a typical U-speed cell:
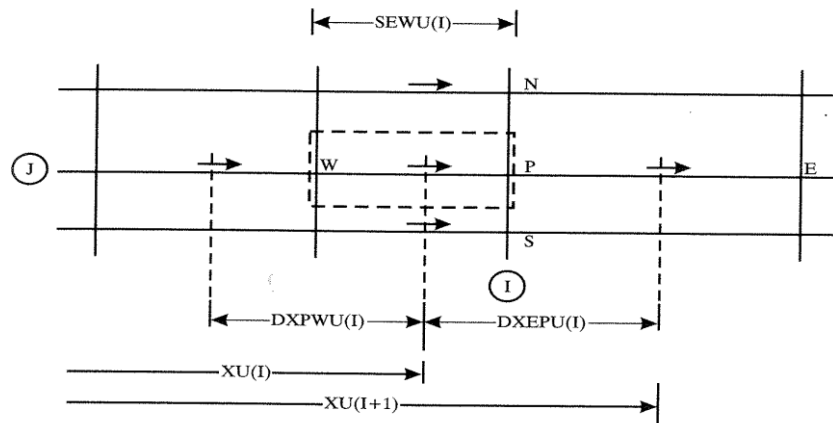
Let's note that the western limit of the cell is located in the mid-range of DXPWU(I) and similarly the eastern limit in the mid-range of DXEPU(I). The northern and southern limits are located in the mid-range of the distances NP and PS respectively.

The next figure shows the typical V-speed cell with FORTRAN variables:
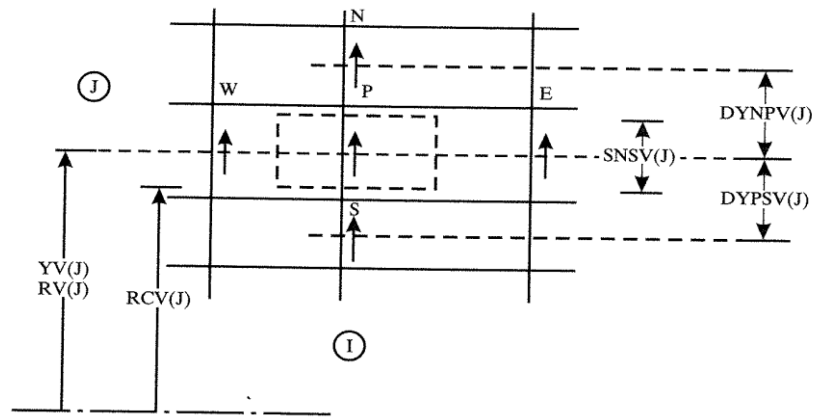
The northern limit of the cell is located in the mid-range of DYNPV(J). Similarly, the southern limit is located in the mid-range of DYPSV(J). The western and eastern limits are located in the mid-range of WP and PE, respectively.

Then, the structure and the functions of the various subroutines of the TEACH-T program are described. In general, each subroutine is subdivided into sections to facilitate the understanding of the entire program. The functions of the various parts of the main subroutine CONTRO are given below:

Section 1: The initial elements of the grid, the control parameters of the program, the constants of the problem and some other related elements are printed outside.

Section 2: The calculations of the grid parameters, the preparation for the supply of the tables (through INIT) and the description of the defined limit values, as well as the preliminary printing of the initial variables are done here.

Section 3: Prepares and checks the repetition and gives intermediate prints of NITER, RESOR, Φ(IMON, JMON) and the distributions of Φ (if NITER is an exact multiple of INDPRI).

Section 4: The final print, the calculations of the shear stress coefficients, the normalization of the distributions and so on, are given here.

The following table gives the general structure of all the CALCΦ subroutines, except CALCP:
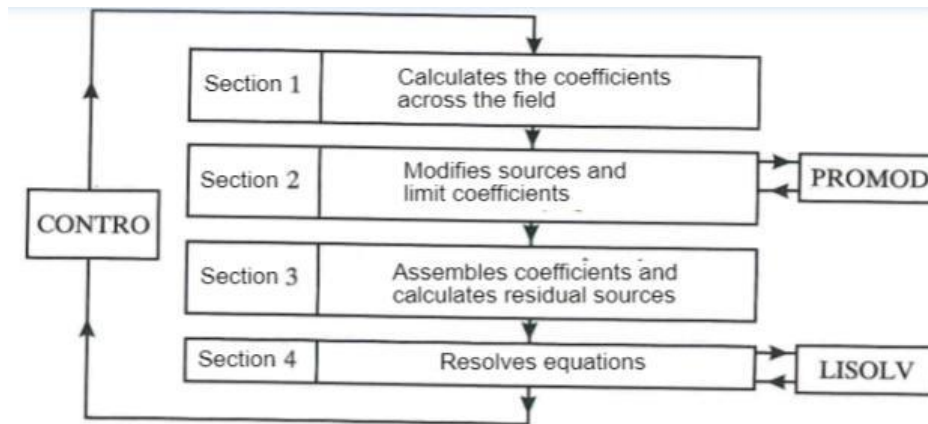
The input and output of each of these subroutines is CONTRO.

Section 1: Calculates the coefficients across the field, using the expressions of the total flow. These calculations are independent of the type of the problem.

Section 2: PROMOD is called, to modify sources and limit coefficients, in order to capture the particular problem.

Section 3: All the coefficients and the RESORΦ residual sources are calculated from Φ of the previous repetition.

Section 4: LISOLV is called to apply the LBL method.

The CALCP subroutine is distinguished from the other CALCΦ subroutines. In Section 1 it has an additional feature. The absolute mass sources are accumulated and stored as RESORM. Residual sources for the P′-equation are not calculated, because they do not provide us with useful information. The P′equation is unlikely to take a unique form, because α_i are unlikely to become zero. Therefore, no special precautions are taken in the introduction of "false" sources. For the CALCP subroutine there is an additional section, **Section 5**, for the calculation of the pressure and speed corrections. In the PROPS subroutine the fluid properties are calculated (thermodynamics and transfer, e.g. μ_{eff}, Γ_{eff}etc.).

The iterative LBL method is performed in the LISOLV subroutine, which has such an arrangement that resolves along the lines N-S with W-E scans. If desired, LISOLV can resolve along the lines W-E with N-S scans. The most important elements of LISOLV is the PHI(I, J) table, which contains the calculated variables and the indicators of the position ISTART and JSTART. The INIT subroutine calculates in **Section 1** the grid coordinates, the distances of the mid-ranges between the nodes, the cell dimensions. R(J) is set equal to 1.0, if INDCOS = 1 (flat flow), or equal to Y(J), if INDCOS = 2 (axis-symmetric flow). In **Section 2** the initial values of the dependent variables of the tables are defined.

The printing of the dependent variables of the tables is performed by the PRINT subroutine. The special elements are: PHI(I, J): table to be calculated; X(I), Y(J): coordinates of storage positions; HEAD: table containing the order of the names of the contained variables; ISTART, JSTART: initial values of the indicators I, J.

In general, the TEACH-T computational program is written in such a way that it is easy to understand and many of its subroutines (except CONTRO, PROPS and PROMOD), do not need modifications for other types of problems.

CHAPTER 6

CONVERSION OF DIFFERENTIAL EQUATIONS INTO FINITE DIFFERENCES AND SYMBOLISM OF VARIABLES IN PROGRAMMING LANGUAGE

6.1   Integration of the Differential Equations of the TEACH-T Variables and their Symbolism in FORTRAN Language

The control volume of the dimensions d_x, d_y, d_z of the figure is considered:
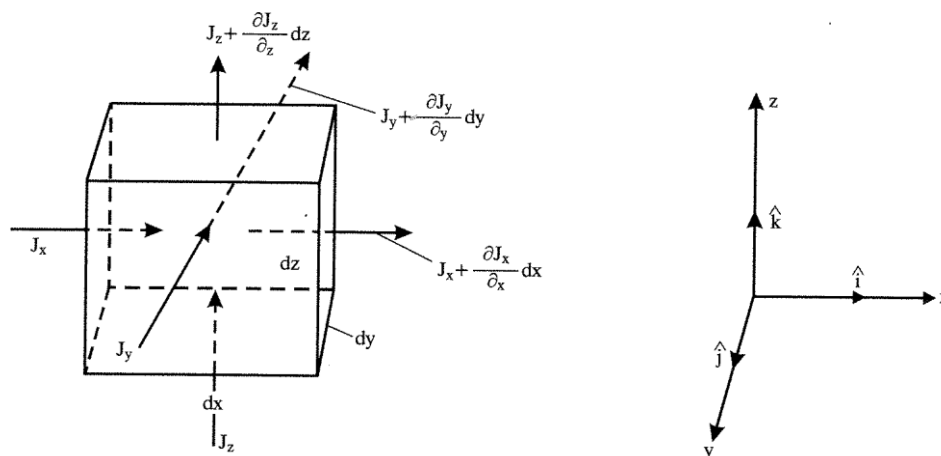
Let's suppose that in the elementary volume dV a quantity expressed by the vector $\vec{J}$ enters

$$\vec{J} = J_x \, \hat{i} + J_y \, \hat{j} + J_z \, \hat{k}$$

As this quantity passes through the volume dV, changes and exits as:

$$J_i + \frac{\partial J_i}{\partial x_i} \, dx_i$$

in the direction i. Then, for the same direction the change of the quantity J will be:

$$(\text{Change of the Ji}) = \frac{\partial J_i}{\partial x_i} \, dx_i$$

If this quantity is reduced in the input surface, then the net outflow of this quantity for each direction separately will be:

$$x \rightarrow \left(\frac{\partial J_x}{\partial x} dx \right) dy\, dz$$

$$y \rightarrow \left(\frac{\partial J_y}{\partial y} dy \right) dx\, dz$$

$$z \rightarrow \left(\frac{\partial J_z}{\partial z} dz \right) dx\, dy$$

This leads to a total outflow from the elementary volume of the quantity J:

$$\text{Outflow} = \left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} \right) dx\, dy\, dz \iff$$

$$\iff \text{Outflow / (volume unit)} = \Sigma_i \frac{\partial J_i}{\partial x_i} = \text{div } \vec{J}$$

Let's also assume that this quantity is of such nature that is kept constant. Then, the outflow of the quantity div$\vec{j}$ will come (if it is not zero) from some other quantity J contained in the control volume dV or/and from some other source J within the volume dV. If the variable of $\vec{j}$ was made in time dt, then the requirement to maintain this quantity is expressed by the following differential equation:

$$\frac{\partial(\varrho J)}{\partial t} + \text{div } \vec{J} = S_j \tag{6.1.1}$$

where ρ: density of J in the volume dV. S_j:source J in the volume dV. The above equation is the general form of the conservation equations. Any conservation equation in physics can be transformed into this form. The continuity equation expresses the principle of the mass conservation, when it enters an elementary volume dV under a flow field of $\vec{u}$ speed:

$$\frac{\partial \varrho}{\partial t} + \text{div } (\varrho\, \vec{u}) = 0 \quad \text{(without mass source)}$$

This equation in cylindrical coordinates takes the form:

$$\frac{\partial \varrho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \varrho u_r) + \frac{1}{r} \frac{\partial (u_\varphi \varrho)}{\partial \varphi} + \frac{\partial (u_z \varrho)}{\partial z} = 0$$

In a constant 2-D flow this equation is converted into:

$$u = u_z$$

$$\frac{1}{r}\frac{\partial}{\partial_r}(r\varrho v) + \frac{\partial(u\varrho)}{\partial z} = 0 \quad \text{where}$$

$$v = u_r$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(r\varrho v) + \frac{\partial}{\partial z}(r\varrho u)\right] = 0 \iff \frac{\partial}{\partial r}(r\varrho v) + \frac{\partial}{\partial z}(r\varrho u) = 0 \qquad (6.1.2)$$

The momentum equations express the momentum conservation in any direction of space. Thus, there are three such equations that have a general form as follows:

$$\frac{\partial}{\partial t}(\varrho u_i) + \text{div}(\varrho \vec{u}\, u_i) = \text{div}(\mu \text{ grad } u_i) + S_i$$

where: $\frac{\partial}{\partial t}(\varrho u_i):$ momentum change in time.

$\text{div}(\varrho \vec{u}\, u_i):$ The convection term that expresses the net momentum outflow

from the elementary volume dV.

$\text{div}(\mu \text{ grad} u_i):$ Diffusion term that expresses the momentum redistribution,

due to viscosity forces.

$S_i:$ momentum source

$i:$ momentum conservation direction.

In cylindrical coordinates in 2D constant flow, this equation is written as follows:

$$\frac{1}{r}\left\{\frac{\partial}{\partial z}(\varrho u\, r u_i) + \frac{\partial}{\partial r}(\varrho v\, r u_i) - \frac{\partial}{\partial z}\left(r\mu \frac{\partial u_i}{\partial z}\right) - \frac{\partial}{\partial r}\left(r\mu \frac{\partial u_i}{\partial r}\right)\right\} = S_i \qquad (6.1.3)$$

In this form the momentum equation is used in the TEACH program. The term of the sources is expressed by the following relations:

a) Momentum equation in the x-direction:

$$Su = \frac{1}{r}\frac{\partial}{\partial_r}\left(r\mu \frac{\partial v}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial x}\left(r\mu \frac{\partial u}{\partial x}\right) - \frac{\partial P}{\partial z} - \frac{2}{3}\frac{\partial}{\partial x}(\mu \text{ div } \vec{u}) \qquad (6.1.4)$$

b) Momentum equation in the y-direction:

$$Sv = \frac{\varrho w^2}{r} - \frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial x}\left(\mu r\frac{\partial u}{\partial r}\right) - \frac{2}{3}\frac{1}{r}\frac{\partial}{\partial r}(r\,\text{div}\,\vec{u}\,\mu) \qquad (6.1.5)$$

We accepted the flow as 2D, but we kept the speed w in the term:

$$\frac{\varrho w^2}{r}$$

In fact, this happens because the flow we have is axis-symmetric, as a result the speed of the coordinate φ is not only different at every point in a plane of calculations, but is also constant in any other plane of calculations.

That is, we can imagine a plane of calculations within the cylinder in which the speed w presents a distribution which it maintains for any other similar plane in the combustion chamber.
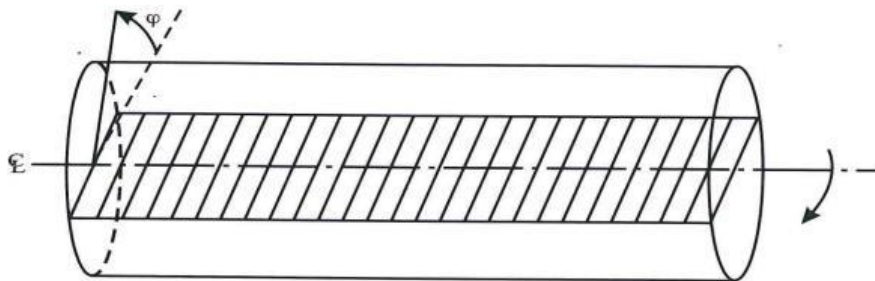

The numerical resolution of the differential equations in TEACH is done by converting them into finite difference equations (FDE) with the method of the control volumes. According to this method, the differential equations are completed in a fairly small control volume and are thus expressed in difference equations. Previously, the calculation space is analyzed in small control volumes. A typical control volume for the TEACH cylindrical combustion chamber is shown in the following figure:
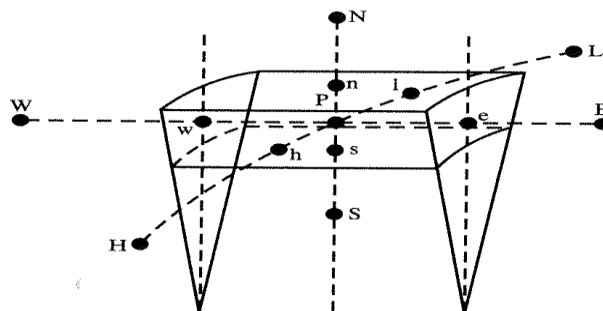
The center of the control volume is the point P, while the neighboring centers of the neighboring control volumes are: W, E, N, S, H, L. The intersection points of the axes connecting these centers with P, with the surfaces of the control volume are, respectively, w, e, n, s, h, l.

In the TEACH program, because we consider the field as axis-symmetric, we do not examine the points H, L. We also consider that the control volume has an angle φ equal to one (φ = 1). Thus, we can see this volume in the plane, as shown in the following figure:
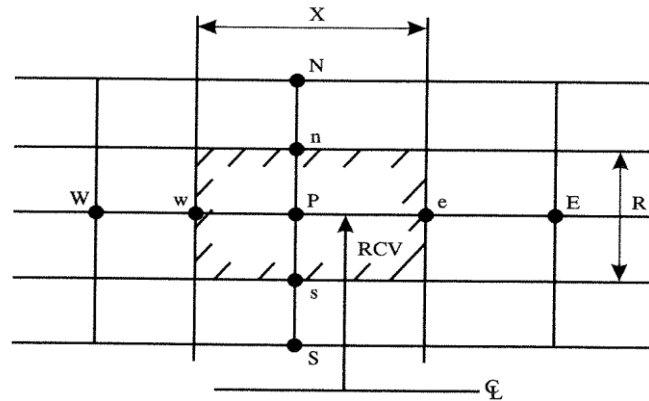
The control volume which in the program is symbolized by VOL, is equal to:

$$VOL = R \, X \, (RCV\varphi) = R \, X \, RCV$$

based on the view of φ = 1. We complete the differential equations not on surfaces dxdy, but in the volume dv, where: dv = r dφ dr dx, although we refer to a 2D flow and we use 2D equations. The integration of the differential equations eventually leads to the storage of all the variables (pressure, speed etc.) in the center of the control volume P. However, it has been proven that this can lead to computational errors, and especially to the definition of the term $-(\partial P/\partial x_i)$ in the momentum equations. For this reason a variant of the above method is used. The new method requires the use of different control volumes to complete the momentum equations in relation to a basic control volume. Thus, the speed u is calculated by completing the momentum equation in the x (or z) direction, in a grid of control volumes shifted opposite to this direction, i.e. to the left. Similarly, v is calculated in a downward shifted control volume. On the contrary, the pressure and other characteristic magnitudes of the flow, such as the density, the viscosity etc., are stored in the basic grid of P. The figures (6.1.1) and (6.1.2) show the shifted grids for calculating the speeds u and v by symbolizing their dimensions, according to the TEACH program. Some dimensions of the grids may give the impression that they are left over. This happens because the grids are located in the middle of the distances of the nodes P, N, S, E, W. We have, however, the option of choosing a non-uniform grid. Understanding the geometry of the grids is very important, in order to understand the integrations of the differential equations. In particular, attention must be paid to the complementary grids u, v of the grids v, u, respectively, and to how at the ends of the grids v, u speeds of a direction other than the one being

completed are located. Thus, in the grid of the u-speed, at its four ends, speeds v of the peripheral grids of type v are defined and vice versa.

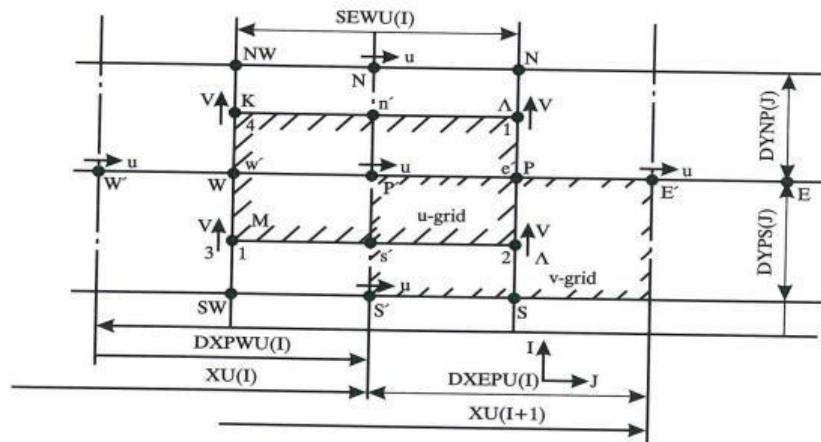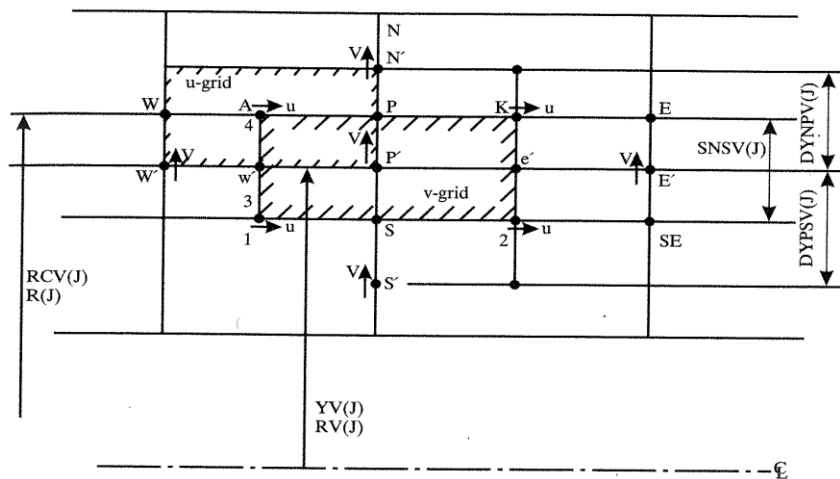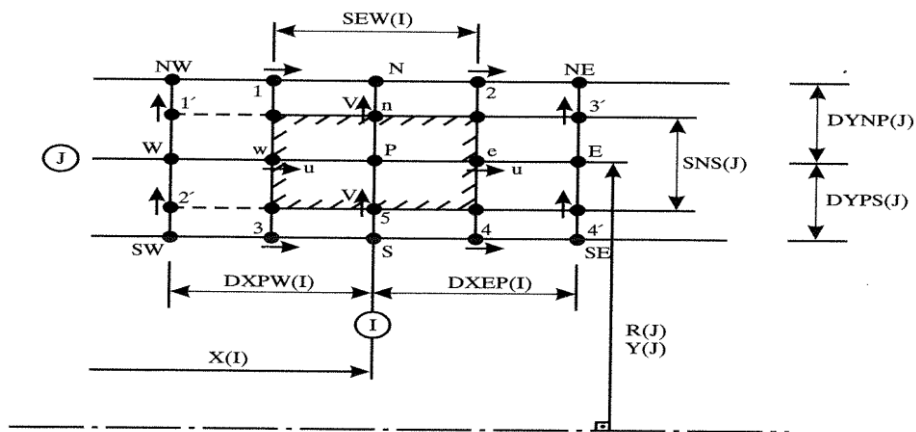**Figure 6.1.1** Control volume for the speed u.

Figure 6.1.2 Control volume for the speed v.

The grids are calculated in the subroutine INIT, which undertakes the zeroing of all the variables which will be calculated in the continuation of the program and the calculation of the geometric sizes of the grids. Initially, a check is made for the value of the indicator INDCOS which determines the type of the coordinates to be used:

$$INDCOS = \begin{cases} 1 & \text{Cartesian Coordinate System (CCS)} \\ 2 & \text{Cylindrical Coordinate System (CLCS)} \end{cases}$$

In case we have (CCS), the value of the spoke R(J) becomes one since here, instead of it, we have the distance Y(J). Before we proceed, however, we should say that the distances Y(J) and X(I) of the grid have been calculated at the beginning of the program. The horizontal position of P is determined by X(I) and the distance between the previous one in the grid P and the current one is defined by DXPW(I), while its distance from the next one is defined by DXEP(I). Therefore, if we define DXEP(I) as:

DXEP(I) = X(I+1) − X(I)

it is obvious that we will have:

DXPW(I+1) = DXEP(I)

We also set values for the grid limit positions expressed on the axis x: (1) = first position, NI = last position. The limit values will obviously be:

DXPW(1) = 0 and DXEP(NI) = 0

The same applies to the vertical distances DYNP(J) and DYPS(J) which become:

DYPS(J+1) = DYNP(J) where DYNP(J) = Y(J+1) − Y(J)

with limit values:

DYPS(1) = 0, DYNP(NJ) = 0

since we have defined on the axis y: (1) = first position, NJ = last position.

Having determined these distances from the figure, it is obvious that:

SEW(I) = 1/2[DXEP(I) + DXPW(I)]

with limit values:

SEW(1) = 0 and SEW(NI) = 0

Likewise, for the y-direction we will now have:

SNS(J) = 1/2[DYNP(J) + DYPS(J)]

with limit values:

SNS(1) = 0 and SNS(NJ) = 0

Then, based on the figure (6.1.1), we calculate the grid geometry of the speed U. The figure shows:

XU(I) = 1/2[X(I) + X(I – 1)]

In this way, we place the center P´for the grid U in the middle between the points P and W. This is very important for the calculations of the momentum-continuity equations in the grid. Then, we calculate, as we did for the P-grid, the distances between the successive P points of the control volumes. Thus, it follows:

DXEP(I) = XU(I + 1) – XU(I)
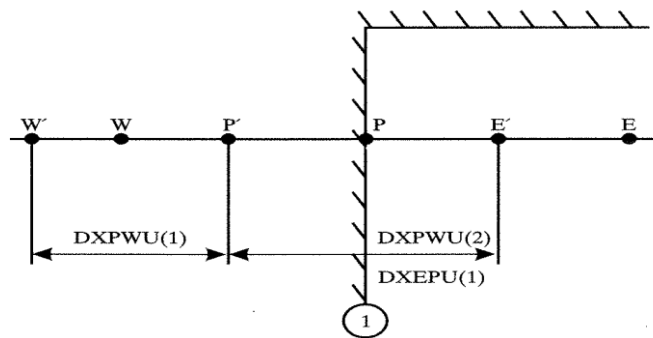
DXPWU(I + 1) = DXEPU(I)

The limit values will be:

DXPWU(1) = 0 and DXPWU(2) = 0

DXEPU(1) = 0 and DXEPU(NI) = 0

The following figure shows the above limit conditions:

From the figure (6.1.1) we can calculate:

SEWU(I) = X(I) – X(I – 1) with limit value SEWU(1) = 0

In the V-grid (see figure (6.1.2)), it will be valid for the spokes RV(J) or YV(J) in CCS:

RV(J) = 1/2[R(J) + R(J – 1)]

YV(J) = 1/2[Y(J) + (J − 1)]

with limit value YV(1) = RV(1) = 0

In this way, the center of the V-cell is placed in the middle of the distance PS. Then, the spoke of the top line of the V-cell is calculated, which otherwise, would not be equal to R(J) based on the following relation:

RCV(J) = 1/2[RV(J + 1) + RV(J)]

with limit values: RCV(1) = R(1) and RCV(NJ) = R(NJ)

Thus, we can now calculate the vertical distances between the successive cells of type V, as we did for the horizontal distances in the U-cell as follows:

DYNPV(J) = YV(J + 1) − YV(J)

DYPSV(J + 1) = DYNPV(J)

with limit values:

DYPSV(1) = 0 and DYPSV(2) = 0

DYNPV(NJ) = 0

Finally, we calculate some geometric coefficients for the u and v cells that there is no point mentioning here, since we will meet them when we need them below in completing the differential equations.

For the calculation of u (CALCU), the momentum equation is written again in the x-direction (or z):

$$\frac{1}{r}\left[\frac{\partial}{\partial x}\left((\varrho u)\, ru\right) + \frac{\partial}{\partial r}\left((\varrho v)\, ru\right)\right] - \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)\right] = S_u \qquad (6.1.6)$$

I   II

Convection terms   Diffusion terms

Then, we complete each term in the control volume of the u grid

Convection terms:

$$\iiint \frac{1}{r}\frac{\partial}{\partial x}(\varrho u\, ru)\, r\, d\varphi\, dx\, dr = \int_{r=3}^{r=4}\int_{x=1}^{x=2}\frac{\partial}{\partial x}(\varrho u\, ru)\, dx\, dr$$

$$\Rightarrow \int_3^4 (\varrho u)\, ru\big|_1^2\, dr = \int_3^4 (\varrho u)\, ru\,\big|_2\, dr - \int_3^4 (\varrho u)\, ru\,\big]_1\, dr$$

(a)   (b)

Line 2 at the point P refers to the magnitude $(\varrho u)/2$ to be calculated. At P, ρ is defined, but u is not defined. Thus, we need to get some average value for this. Due to the field geometry we will have:

$$u_P = u'_E \frac{SEWU(I)}{SEWU(I) + SEWU(I+1)} + u'_P \frac{SEWU(I+1)}{SEWU(I) + SEWU(I+1)}$$

Let's not forget that u are stored only at the points (P′, N′, S′, W′). To simplify the calculations, the program in the subroutine INIT determines the geometric coefficient:

$$WFE(I) = \frac{SEWU(I)}{SEWU(I) + SEWU(I+1)}$$

therefore:

$$u_P = u'_E \left(1 - WFE(I)\right) + u'_P \, WFE(I)$$

or with speed indicators

$$u_P = u \, (I+1, J) \left(1 - WFE(I)\right) + u \, (I, J) \, WFE(I)$$

The density at P is symbolized by DEN(I,J) so the term $(\varrho u)/2$ will be equal to: $u_P DEN(I,J)$

The convection speed u is none other than u_P. The term $dr|_3^4$ expresses the length between the lines 3-4 and according to the shapes of the grids is the term SNS(J). r again from the grids is RCV(J), so we can calculate the surface:

AREAEW = SNS(J) RCV(J)

For simplification (according to the program) we set:

GE = DEN(I,J) (U(I + 1,J) (1 − WFE(I)) + U(I,J)WFE(I))

CE = AREAEW GE

as a result this term is written as follows:

Convection E= $CE u_P$ =CE(U(I,J) WFE(I)+U(I+1,J) (1-WFE(I)))

where u_P is the convection speed. In line 1, at the point W, where the other integral refers, things are exactly the same. The indicators are simply shifted one position to the left. Thus, while the term $r \, dr|_3^4$ remains AREAEW, the density ρ_w or DEN(I-1,J) and the speed u_w is calculated as follows:

$$u_w = U(I, J)WFE(I) + U(I-1, J)(1-WFE(I))$$

where:

$$WFW(I) = \frac{SEWU(I-1)}{SEWU(I) + SEWU(I-1)}$$

We set the coefficients:

GW = DEN(I-1,J) (U(I,J) WFW(I)=U(I-1,J) (1-WFW(I)))

CW = GW AREAEW

as a result the integral is written as follows:

Convection W=CW U_w=CW(U(I,J) WFW(I)+U(I-1,J) (1-WFW(I)))

where U_w is the convection speed.

$$\int_3^4 \int_1^2 \frac{\partial}{\partial r} ((\varrho v)\, ru)\, dr\, dx = \int_1^2 (\varrho v)\, ru \left.\right|_3^4 dx =$$

$$= \int_1^2 (\varrho v)\, ru \rfloor_4\, dx - \int_1^2 (\varrho v)\, ru \rfloor_3\, dx$$

       (a)   (b)

a) The term $(\varrho v \rfloor_4)$ refers to the point n´. Here this term cannot be defined at this point, since neither ρ nor v is stored at this point. Thus, we must necessarily set an average value for this term. Let's see which points are around n´:

N´, N, P, P´, W, K, Λ, NW.

We observe that the density is stored at the points N, P, W, NW, while the speed v at the points K, Λ. Therefore, the average value will be calculated based on these points. Based on the above it is obvious that the following expression is an acceptable average value of the quantity (ρV) at n´.

$$\left[\frac{1}{2}\left(\varrho_{NW} + \varrho_W\right)V_\varkappa + \frac{1}{2}\left(\varrho_N + \varrho_P\right)V_\Lambda\right] \Big/ 2$$

The speed V can only be stored at the ends of the control volume of U. This magnitude with the program symbols is written as follows:

0,25 [(DEN(I-1, J+1) + DEN(I-1, J)) V(I-1, J+1) +

+ (DEN(I, J+1) + DEN(I,J)) V(I, J+1)]   (6.1.7)

We repeat that in the figure (6.1.1) U(I,J) is stored at the point P´, while V(I,J) at the point A, although the indicators are the same (The coordinates I, J do not refer to the same point, when expressing the positions of different magnitudes). In fact, we have a series of overlapping grids that each one of them has its own coordinates I, J. In the program there are the coefficients:

$$\mathbf{GN} = 0.5 \left(\varrho_N + \varrho_P \right) \mathbf{V} \, (\mathbf{I, J} + 1)$$

$$\mathbf{GNW} = 0.5 \left(\varrho_{NW} + \varrho_W \right) \mathbf{V} (\mathbf{I} - 1, \mathbf{J} + 1)$$

The term $\mathrm{dx}\rfloor_1^2$ obviously becomes SEWU(I), while the spoke r at the height of the line 4 is RV(J+1) and is directed to the speed V. Thus, we can set:

AREAN = RV(J+1)SEWU(I)

as a result the integral becomes:

$$0.5(\mathbf{GN} + \mathbf{GNW})\mathbf{AREAN} \; \mathbf{U'_n}$$

The term 0.5 (GN + GNW) = CN is the term of the relation (6.1.7). It remains, therefore, to determine the speed u´n. This speed, which is the convection speed, is defined at the points N´ and P´. Because the lengths N´n´and P´n´are equal, the average value of the speed u at n´can be defined as:

$$\mathbf{u'_n} = \frac{\mathbf{u'_N} + \mathbf{u'_P}}{2} = 0.5 \, (\mathbf{U}(\mathbf{I} - 1, \mathbf{J} + 1) + \mathbf{U}(\mathbf{I, J}))$$

In this way, the integral is written as follows:

$$\mathrm{Convection} \; \mathbf{N} = 0.5 \; \mathbf{CN} \; (\mathbf{U}(\mathbf{I, J}+1) + \mathbf{U}(\mathbf{I, J}))$$

In this integral the magnitudes are the same, except that the indicators are shifted one position downwards. Thus, the average value of the quantity $(\varrho v)\rfloor_3$ is calculated based on the point B´and is:

$$(\varrho v)\rfloor_3 = \left[\frac{1}{2} \, (\varrho_W + \varrho_{SW}) \, \mathbf{V} \, (\mathbf{I} - 1, \mathbf{J}) + \frac{1}{2} \, (\varrho_S + \varrho_P) \, \mathbf{V} \, (\mathbf{I, J}) \right] \Big/ 2$$

so, based on the program symbols, it is valid:

GS = 0.5 (DEN(I, J-1) + DEN(I, J))V(I,J)

GSW = 0.5 (DEN(I-1, J) + DEN (I-1, J-1))V(I-1, J)

The magnitude $dx\rfloor_1^2$ is SEWU(I), while the spoke r now refers to the line 3 and is symbolized by RV(J). Therefore, the integral is written as follows:

$$0.5 \ (GS+GSW) \ AREAS \ U'_S$$

where: AREAS = RV(J) SEWU(I). The speed U´$_S$ is calculated similarly to U´$_n$ as:

$$U'_S = \frac{U'_P + U'_S}{2} = 0.5 \ (U(I, J) + U \ (I, J - 1))$$

We also set GS = 0.5(GS+GSW). Thus, the integral finally takes the form:

$$Convection \ S = 0.5 \ CS(U(I, J)+U(I, J-1))$$

Diffusion terms:

$$\iiint \frac{1}{r} \frac{\partial}{\partial x}\left(r\mu \frac{\partial u}{\partial x}\right)r \ d\varphi \ dx \ dr = \int_{r=3}^{r=4}\int_{x=1}^{x=2} \frac{\partial}{\partial x}\left(r\mu \frac{\partial u}{\partial x}\right)dx \ dr =$$

$$= \int_3^4 r\mu \frac{\partial u}{\partial x}\rfloor_2 \ dr - \int_3^4 r\mu \frac{\partial u}{\partial x}\rfloor_1 \ dr$$

$\left(r\mu \dfrac{\partial u}{\partial x}\right)\!\diagup 2$ refers to the point P. There, the viscosity is defined as μ$_P$ = VIS(I,J). The spoke r is expressed by the magnitude RCV(J), while $dr\rfloor_4^3$ by the length SNS(J). It remains to calculate the term $(\partial u/\partial x)\rfloor_2.$ u is stored at the nearby points P´and E´and therefore it can be written as:

$$\frac{\partial u}{\partial x}\rfloor_2 = \frac{U'_E - U'_P}{\Delta x_{E'P'}} = \frac{U \ (I + 1, J) - U \ (I, J)}{DXEPU \ (I)}$$

Setting AREAEW = SNS(J) RCV(J) and DE = (VIS(I, J) AREAEW)/ DXEPU(I) the integral takes the form:

$$Diffusion \ E = DE(U(I+1, J) - U(I, J))$$

We refer to the point W in a similar way, so we have:

$$\mu_w = VIS(I\text{-}1, J), \qquad AREAEW = RCV(J) \ SNS(J)$$

$$\frac{\partial u}{\partial x}\rfloor_1 = \frac{U'_P - U'_W}{\Delta x_{P'W'}} = \frac{U \ (I, J) - U \ (I - 1, J)}{DXPWU \ (I)}$$

We name DW = (VIS(I-1, J)AREAEW) /DXPWU(I) and it follows:

Diffusion W = DW(U(I, J) - U(I-1, J))

$$\int_3^4 \int_1^2 \frac{\partial}{\partial r}\left(r\mu\,\frac{\partial u}{\partial r}\right) dx\, dr = \int_1^2 \left(r\mu\,\frac{\partial u}{\partial r}\right)\Big|_3^4 dx =$$

$$= \int_1^2 r\mu\,\frac{\partial u}{\partial r}\Big|_4 dx - \int_1^2 r\mu\,\frac{\partial u}{\partial r}\Big|_3 dx$$

(a) (b)

The magnitude $r\mu\,\dfrac{\partial u}{\partial r}\Big|_4$ refers to the point n′. Here, however, we cannot define the viscosity. So we get an average value for it with the help of the surrounding points N, NW, W, P. Thus, we will have:

$$\mu'_n = 0.25\,(\mu_P + \mu_N + \mu_{NW} + \mu_W)$$

and with the program symbols:

VISN = 0.25 (VIS(I,J)+VIS(I, J+1)+VIS(I-1, J)+VIS(I-1, J+1))

The term:

$$\frac{\partial u}{\partial r}\Big|_4 = \frac{U'_N - U'_P}{\Delta r_{N'P'}} = \frac{U\,(I, J+1) - U\,(I, J)}{DYNP\,(J)}$$

The term $dx\Big|_1^2$ = SEWU(I) and r in the position where the above magnitudes are calculated is RV(J+1). We set:

AREAN-RV(J+1) SEWU(I),   DN = (VISN AREAN)/DYNP(I)

so the integral becomes: **Diffusion = DN(I, J+1) - U(I ,J)).**

The magnitudes are calculated similarly to the previous integral with the difference that they now refer to the point s′. Thus, we will have:

AREAS = RV(J) SEWU(I)

$$\frac{\partial u}{\partial r}\Big|_3 = \frac{U_{P'} - U_{S'}}{\Delta r_{P'S'}} = \frac{U\,(I, J) - U\,(I, J-1)}{DYPS\,(J)}$$

We set:

$$VISS = \mu_{B'} = 0.25\left(\mu_P + \mu_S + \mu_{SW} + \mu_W\right) =$$
$$= 0.25\,(VIS(I, J) + VIS(I, J-1) + VIS(I-1, J-1) + VIS(I-1, J))$$

we find the average value of μ at B′. Finally, we set:

DS = VISS AREAS/DYPS(J)

as a result the integral becomes:

Diffusion S = DS (U(I, J)-U(I, J-1))

Source Terms:

According to the equation (6.1.4) the source terms for the momentum equation in this direction are:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial V}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial z}\left(r\mu\frac{\partial u}{\partial z}\right)-\frac{\partial P}{\partial z}-\frac{2}{3}\frac{\partial}{\partial z}(\mu \operatorname{div}\vec{u}) \tag{6.1.8}$$

A different method is followed to convert these terms into difference equations. Because these terms express sources within the control volume, they will also have a constant value inside it always in relation to its dimensions. Thus, we will convert the equations into FDE and then we will multiply the result by the control volume. Within this volume the density is kept constant at all times, since the flow is constant, resulting in divu = 0. Thus, the last term of (6.1.8) leaves. We can, therefore, have:

$$-\frac{\partial P}{\partial z}=-\frac{P_P-P_W}{\Delta xPW}=\frac{P_W-P_P}{SEWU(I)}=\frac{P(I-1, J)-P(I, J)}{SEWU(I)}$$

The control volume is:

VOL = RCV(J) SEWU(I) SNS(J)

Thus, we have:

$$\begin{aligned}(P(I-1, J) - P(I, J))/SEWU(I)\ RCV(J)\ SEWU(I)\ SNS(J) &= \\ = (P(I-1, J) - P(I, J))/SEWU(I)\ SEWU(I)\ RCV(J)\ SNS(J) &= \\ = [P(I-1, J) - P(I, J)]\ AREAEW.\end{aligned}$$

In the program DU(I,J) = AREAEW is set so this term is written as:
DU(I ,J)[P(I-1, J) - P(I, J)].

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial V}{\partial z}\right)=\frac{1}{r}\frac{r\mu\frac{\partial V}{\partial z}\Big|_{n'} - r\mu\frac{\partial V}{\partial z}\Big|_{S'}}{\Delta rn'S'}$$

The term $\frac{\partial V}{\partial z}\Big|_{n'}$ or $\frac{\partial V}{\partial x}\Big|_{n'}$ is written:

$$\frac{\partial V}{\partial x}\Big|_{n'}=\frac{V_A-V_K}{\Delta xKA}=\frac{V(I, J+1)-V(I-1, J+1)}{SEWU(I)}$$

and the other takes the form:

$$\frac{\partial V}{\partial x}\Big|_{S'} = \frac{V_A - V_M}{\Delta x AM} = \frac{V(I, J) - V(I-1, J)}{SEWU(I)}$$

These terms are described as DVDXN and DVDXS, respectively. Thus, finally we can have:

$$1/r\,[DVDXN\,r\mu\big|_{n'} - DVDXS\,r\mu\big|_{s'}]/DVNPV(J) =$$

$$= 1/RCV(J)\,(DVDXN\,RV(J+1)\,VISN - DVDXS\,RV(J)\,VISS)/DYNPV(J)$$

This term is called SORCE2.

$$\frac{1}{r}\frac{\partial}{\partial x}\left(r\mu\,\frac{\partial u}{\partial x}\right) = \frac{1}{r}\,\frac{r\mu\frac{\partial u}{\partial x}\big|_P - r\mu\frac{\partial u}{\partial x}\big|_w}{SEWU(I)}$$

Here r remain constant so they leave, while $\partial u/\partial x$ are:

$$\frac{\partial u}{\partial x}\Big|_P = \frac{U_{E'} - U_{P'}}{\Delta r E' P'} = \frac{U(I+1, J) - U(I, J)}{DXEPU(I)}$$

which is called DUDXE.

$$\frac{\partial u}{\partial x}\Big|_w = \frac{U_{P'} - U_{W'}}{\Delta x\,P'W'} = \frac{U(I, J) - U(I-1, J)}{DXPWU(I)}$$

which is called DUDXW. Thus, we will have:

$$SORCE2 = (VIS(I, J)\,DUDXE - VIS(I-1, J)\,DUDXW)/SEWU(I)$$

Therefore, we have source terms:

$$Source = DU(I, J)\,(P(I-1, J) - P(I,J))) + SORCE1\,VOL + SORCE2\,VOL$$

Composing the equations for CALCU, it follows that:

[Convection N - Convenction S] + [Convection E - Convection W] -
- [Diffusion N - Diffusion S] - [Diffusion E - Diffusion W]= Source →

→ CE(U(I, J)WFE(I) + U(I+1, J)(1 - WFE(I))) - CW(U(I,J)WFW(I) + U(I-1, J)(1 - WFW(I)))+ 0,5 CN(U(I, J+1) +U(I, J) - 0.5 CS(U(I,J) + U(I, J-1)) - DE(U(I + 1,J) - U(I,J)) + DW(U(I, J) - U(I-1, J)) - DN(U(I, J + 1) - U(I,J)) + DS(U(I, J) - U(I, J-1)) = DU(I, J)(P(I-1, J) - P(I, J)) + SORCE1VOL + SORCE2 VOL

→ V(I, J)[CEWFE(I) - CWWFW(I) + 0.5 CN - 0.5 CS + DE + DW + DN + DS] = U[(I- 1) [CW(1 - WFW(I)) + DW] + U(I + 1,J) [DE - CE (1 -WFW(I)] + U[I, J-1)] [0.5 CS + DS] + U(I, J+1)[DN - 0.5CN] + DU(I, J)(P(I-1, J) - P(I, J)) + SORCE1 VOL + SORCE2 VOL

we set:

AW = CW(1 - WFW(I)) + DW

AE = DE - CE(1 - WFE(I))

AS = 0.5CS + DS

AN = DN - 0.5CN

A_p = CEWFE(I) – CWWFW(I) + 0.5CN + DE – 0.5CS + DW + DN + DS

Calculation of V(CALCV):

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(\varrho urV) + \frac{\partial}{\partial r}(\varrho VrV)\right] - \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\mu\frac{\partial V}{\partial x}\right) - \frac{\partial}{\partial r}\left(r\mu\frac{\partial V}{\partial r}\right)\right] = S_V \qquad (6.1.9)$$

(I)convection terms   (II)diffusion terms

Then, we complete each term separately, following the logic of CALCU.

Convection terms

$$\iiint_{Vv} \frac{1}{r}\frac{\partial}{\partial x}(\varrho urV)\, dx\, rd\varphi\, dr = \int_3^4 (\varrho urV)\Big|_1^2 dr =$$

$$\int_3^4 \varrho urV\Big|_2\, dr - \int_3^4 \varrho urV\Big|_1\, dr$$

(a)   (b)

a) On line 2, at the point e´, the value of the convection magnitude (ρu) will be calculated as the average value at the surrounding points. The density ρ is stored at the points P, E, S and SE, while the speed u at the points K, Λ and therefore we can write:

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$$(\varrho u)_P = [1/2\,(\varrho_{SE} + \varrho_S)\,U(I + I, J - I) + 1/2\,(P_p + P_e)\,U\,(I + I, J)]/2$$
$$\qquad\qquad (U_A)\qquad\qquad\qquad\qquad\qquad (U_K)$$

The term $dr|_3^4$,while on this line, r = RV(J), so we can set:

AREAEW = SNSV(J) RV(J)

So finally with the program symbols we can write:

GE = 0.5 (DEN(I, J) + DEN(I+1, J))U(I+1, J))
GSE = 0.5(DEN(I+1, J-1) + DEN(I, J-1))U(I+1, J-1)

Because the points A, K were placed in the middle of the lines WP and PE respectively, the speed V at e´is defined as follows:

$$V_{e'} = \left(V_{E'} + V_{P'}\right)/2 + 0.5(V(I + 1, J) + V(I, J))$$

Thus, finally the integral takes the form:

Convection E = 0.5CE (V(I+1, J) + V(I, J))

where CE = 0.5 (CE + GSW) AREAEW

The calculation for this term is similar, except that the indicator I becomes I-1. Thus, we will have:

GW = 0.5(DEN(I, J)+DEN(I-1, J))U(I, J)
GSW = 0.5(DEN(I-1, J-1)+DEN(I, J-1)) U(I, J-1)
CW = 0.5(GW+GSW)AREAEW

The speed V at W will be equal to:

$$V_{w'} = \left(V_{W'} + V_{P'}\right)/2 = 0.5(V(I, J) + V(I - 1, J))$$

and:

Convection W = 0.5 CW(V(I, J)+V(I-1, J))

$$\int_3^4 \int_1^2 \frac{\partial}{\partial r}\,((\varrho V)\,rV)\,dx\,dr = \int_1^2 ((\varrho V)\,rV)\,|_3^4\,dx =$$

$$= \int_1^2 (\varrho V)\,rV\,|_4\,dx - \int_1^2 (\varrho V)\,rV\,|_3\,dx$$

where ρ is defined, so it becomes:

$$\varrho_P = DEN(I, J)$$

(a)　　　　　　　　　(b)

We refer to the point P,

On the contrary, V is not defined and it will take an average value between the points P´ and N´ at which it is also defined: due to the grid geometry it will be valid:

$$V_P(SNSV(J) + SNSV(J + 1)) = \underset{\downarrow}{V(I, J)\,(SNSV(J + 1))} + \underset{\downarrow}{V(I, J + 1)(SNSV(J))}$$
$$\qquad\qquad\qquad\qquad\qquad\quad V'_P \qquad\qquad\qquad\qquad V'_N$$

$$\rightarrow V_P = V(I,J)SNSV(I+1)/(SNSV(J+1)+SNSV(J))+$$
$$V(I,J+1)SNSV(J)/(SNSV(J+1)+SNSV(J))$$

In the subroutine INIT the geometric coefficient has been calculated:

$$WFN(J) = SNSV(J + 1)/(SNSV(J) + SNSV(J+1))$$

so we can write V_P in the following way:

$$V_P = V(I,J + 1)(1 - WFN(J)) + V(I,J)WFN(J)$$

$dx\rfloor_1^2$ = SEW(I) while r at this point is R(J), so we immediately define the surface as follows:

AREAN = R(J) SEW(I)

Then we set GN = VP DEN(I,J) and

CN = GN AREAN

The convection speed V is also expressed by the equation:

$$V_P = V(I,J + 1)(1 - WFN(J)) + V(I,J)WFN(J)$$

as a result it is defined as:

Convection N = CN(V(I,J+1) (1-WFN(J))+V(I,J) WFN(J))

This term refers to the line 3 at the point S and is thus it is the same as the previous one by modifying J by one (J-1). Thus, we can have:

PS = DEN(I, J-1)

Here r is R(J-1), therefore the surface is equal to:

AREAS = (R(J-1) SEW(I)

The speed at S is written as follows:

$$V_s = V(I,J)WFS(J)+V(I,J-1)(1-WFS(J))$$

where:

$$WFS(J) = SNSV(J-1)/(SNSV(J) + SNSV(J-1))$$

and is defined in the subroutine INIT.

We set:

$$GS = DEN(I,J-1)V_S \qquad \text{and} \qquad CS = GS\ AREAS$$

The convection speed V is the same as that expressed by the equation:

$$V_S = V(I,J)WFS(J) + V(I,J-1)(1-WFS(J))$$ and thus the integral is written as:

$$\text{Convection } S = CS(V(I,J)\ WFS(J) + V(I,J-1)\ (1-WFS(J)))$$

Diffusion terms:

$$\iiint_V \frac{1}{r} \frac{\partial}{\partial x}\left(r\mu \frac{\partial V}{\partial x}\right) r\, d\varphi\, dx\, dr = \int_3^4 \int_1^2 \frac{\partial}{\partial x}\left(r\mu \frac{\partial V}{\partial x}\right) dx\, dr =$$

$$= \int_3^4 r\mu \frac{\partial V}{\partial x}\Big|_2 dr - \int_3^4 r\mu \frac{\partial V}{\partial x}\Big|_1 dr$$

(a) (b)

On line 2, at the point e´, r = RV(J), the term $dr\big|_3^4$ = SNSV(J), while the viscosity will be calculated as the average value of the surrounding values, since it is not stored at e´, but at P, E, SE, S. Thus, it arises:

$$\mu\big|_{e'} = VISE = 0.25(VIS(I,J) + VIS(I+1,J) + VIS(I,J-1) + VIS(I+1,J-1))$$

We set AREAEW = RV(J) SNSV(J)

The derivative $\dfrac{\partial V}{\partial x}\Big|_2$ is calculated as follows:

$$\frac{\partial V}{\partial x}\Big|_2 = \frac{\partial V}{\partial x}\Big|_{e'} = \frac{V_{E'} - V_{P'}}{\Delta x_{E'P'}} = \frac{V(I+1,J) - V(I,J)}{DXEP(I)}$$

Setting:

$$DE = VISE\ AREAEW/DXEP(I)$$

the integral will take the form:

$$\text{Diffusion } E = DE(V(I+1,J) - V(I,J))$$

On line 1, at the point w′, the calculation is the same as the position where I, I-1.

$$\mu\rfloor_{w'} = VISW = 0.25(VIS(I,J) + VIS(I,J-1) + VIS(I-1,J) + VIS(I-1,J-1))$$

$$AREAEW = R(J)\ SNSV(J)$$

The term $\dfrac{\partial V}{\partial x}\rfloor_1$ takes the form:

$$\frac{\partial V}{\partial x}\rfloor_{w'} = \frac{V_{P'} - V_{W'}}{\Delta x_{P'W'}} = \frac{V(I,J) - V(I-1,J)}{DXPW(I)}$$

Thus, we set:

$$DW = VISW\ AREAEW/DXPW(I)$$

and the integral is written as follows:

Diffusion W = DW (V(I,J) -V(I-1,J))

$$\int_3^4\int_1^2 \frac{\partial}{\partial r}\left(r\mu\frac{\partial V}{\partial r}\right)dx\,dr = \int_1^2\left(r\mu\frac{\partial V}{\partial r}\right)\rfloor_3^4 dx =$$

$$= \int_1^2 r\mu\frac{\partial V}{\partial r}\rfloor_4\,dx - \int_1^2 r\mu\frac{\partial V}{\partial r}\rfloor_3\,dx$$

On line 4, at the point P the viscosity μ is stored at P, so we set μ_P = VIS(I,J), while the spoke r = R(J) and:

$$dx\rfloor_3^4 = SEW(I)$$

The term $\dfrac{\partial V}{\partial r}\rfloor_4$ is written as follows:

$$\frac{\partial V}{\partial x}\rfloor_4 = \frac{\partial V}{\partial x}\rfloor_P = \frac{V_{N'} - V_{P'}}{\Delta r_{N'P'}} = \frac{V(I,J+1) - V(I,J)}{DYNPV(J)}$$

We set:

$$AREAN = R(J)*SEW(I)$$

and it results in:

$$DN = VIS(I,J)\ AREAEW/DYNPV(J)$$

Thus, the integral takes the form:

Diffusion N = DN(V(I,J+1) - V(I,J))

On line 3 at the point S the calculation is the same as before, except that the indicator J becomes J-1.

Thus, it will be valid:

$$\frac{\partial V}{\partial r}\Big\rfloor_S = \frac{V_{P'} - V_{S'}}{\Delta r_{P'S'}} = \frac{V(I, J) - V(I, J-1)}{DYPSV(J)}$$

and

$$\mu \rfloor_S = VIS(I, J\text{-}1) \qquad r = R(J\text{-}1)$$

$$AREAS = R(J\text{-}1)\ SEW(I)$$

$$DS = VIS(I,J\text{-}1)\ AREAS)/DYPSV(J)$$

as a result the integral is written as follows:

$$\text{Diffusion } S = DS\ (V(I,J) \text{ - } V(I,J\text{-}1)$$

Source terms:

According to the equation (6.1.5), the following terms are calculated:

$$\frac{\varrho w^2}{r} - \frac{\partial P}{\partial r} + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\mu\,\frac{\partial V}{\partial r}\right)\right) + \frac{1}{r}\frac{\partial}{\partial x}\left(\mu r\,\frac{\partial u}{\partial r}\right)$$

$$\frac{\varrho w^2}{r} = \frac{\varrho w^2}{r}\Big\rfloor_{P'} = \frac{P\rfloor_{P'}\ w^2\rfloor_{P'}}{r\rfloor_{P'}} =$$

$$\frac{1/2(P_P + P_S)\left[1/2(w_P + w_S)\right]^2}{RV(J)}$$

$$= [0.5(DEN(I,J) + DEN(I,J\text{-}1))0.25(W(I,J) + W(I,J\text{-}1))^2]/RV(J) =$$

$$= [0.125(DEN(I,J) + DEN(I, J\text{-}1))(W(I,J) + W(I, J\text{-}1))^2]/RV(J)$$

This result is multiplied by:

$$VOL = RV(J)\ SEW(I)\ SNSV(J)$$

$$-\frac{\partial P}{\partial r} = \frac{\partial P}{\partial r}\Big\rfloor_{P'} = -\frac{P_P - P_S}{\Delta r_{PS}} = \frac{P_S - P_P}{\Delta r_{PS}}$$

$$\frac{P(I,J\text{-}1) - P(I,J)}{SNSV(J)}$$

when multiplied by the volume VOL, it will be:

$$= [P(I,J\text{-}1) \text{ - } P(I,J)]/SNSV(J)\ RV(J)\ SEW(I)\ SNSV(J) =$$

because we have set:

AREAN-R(J) SEW(I)

and:

AREAS = R(J-1) SEW(I)

the magnitude:

$$0.5(AREAN+AREAS) = 0.5SEW(I)(R(J)+R(J+1)) =$$

$$= SEW(I) [(R(J)+R(J-1))/2] = RV(J) SEW(I)$$

is set as:

DV(I,J) = 0.5(AREAN+AREAS)

Thus, we finally have:

$$= \underline{(P(I,J-1) - P(I,J))\ DV(I,J)}$$

Then:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial V}{\partial r}\right)\Big|_P = \frac{1}{r}\Big|_{P'} \frac{\left(r\mu\frac{\partial V}{\partial r}\right)\Big|_P \left(r\mu\frac{\partial V}{\partial r}\right)\Big|_S}{\Delta r_{PS}}$$

$$\frac{1}{RV(J)} \frac{RCV(J)VIS(I,J)\frac{\partial V}{\partial r}\Big|_P - RCV(J-1)VIS(I,J-1)\frac{\partial V}{\partial r}\Big|_S}{SNSV(J)}$$

And the terms $\frac{\partial V}{\partial r}$ will be:

$$\frac{\partial V}{\partial r}\Big|_P = \frac{V_{N'} - V_{P'}}{\Delta r_{N'P'}} = \frac{V(I,J+1) - V(I,J)}{DYNPV(J)} = DVDYN$$

$$\frac{\partial V}{\partial r}\Big|_S = \frac{V_{P'} - V_{S'}}{\Delta r_{P'S'}} = \frac{V(I,J) - V(I,J-1)}{DYPSV(J)} = DVDYS$$

Thus, we have:

SORCE2 = [(VIS(I,J)RCV(J)DVDYN -
          - VIS(I,J-1)RCV(J-1)DVDYS)] /[RV(J) SNSV(J)]

The term:

$$\frac{1}{r}\frac{\partial}{\partial x}\left(\mu r\frac{\partial u}{\partial r}\right)\!\Big\rfloor_{P'} = \frac{1}{r}\Big\rfloor_{P}\,\frac{\left(\mu r\frac{\partial u}{\partial r}\right)\!\rfloor_{e'} - \left(\mu r\frac{\partial u}{\partial r}\right)\!\rfloor_{w'}}{\Delta x_{W'E'}} =$$

$$= \frac{1}{r}\Big\rfloor_{P}\,\frac{r\rfloor_{E'W'P}\left[\left(\mu\frac{\partial u}{\partial r}\right)_{e'} - \left(\mu\frac{\partial u}{\partial r}\right)_{w'}\right]}{\Delta x_{W'E'}} =$$

$$= \frac{VISE\,\frac{\partial u}{\partial r}\rfloor_{e'} - VIW\,\frac{\partial u}{\partial r}\rfloor_{w'}}{DXEPU(I)}$$

where the terms $\dfrac{\partial u}{\partial r}$ are equal to:

$$\frac{\partial u}{\partial r}\Big\rfloor_{e'} = \frac{u_K - u_\Lambda}{\Delta r_{K\Lambda}} = \frac{U(I+1,\,J) - U(I+1,\,J-1)}{SNSV(J)} = DUDYE$$

$$\frac{\partial u}{\partial r}\Big\rfloor_{w'} = \frac{u_A - u_M}{\Delta r_{AM}} = \frac{U(I,\,J) - U(I,\,J-1)}{SNSV(J)} = DUDYW$$

Thus, this term takes the form:

SORCE1 = [VISE DUDYE - VISW DUDYW]/DXEPU(I)

If we name the first term we calculated (with W) SORCEW, it will be valid:

Source = SORCEW VOL+ SORCE1 VOL +
+ SORCE2 VOL + (P(I, J-1)-P(I, J)) DV(I, J)

We finally have:

[Convection N - Convection S] + [Convection E - Convection W] -
- [Diffusion N-Diffusion S] - [Diffusion E-Diffusion W] = Source

and by replacing their equivalents:

0.5CE(V(I+1,J)+V(I,J)) - 0.5CW(V(I,J) + V(I-1,J)) +
+ CN(V(I,J+1)(1-WFN(J)) + V(I,J)WFN(J)) - CS(V(I,J)WFS(J) +

+V(I,J-1)(1-WFS(J)))-DE(V(I+1,J)-V(I,J))+DW(U(I,J)-
-U(I-1,J))-DN(V(I,J+1)-V(I,J))+DS(V(I,J)-V(I,J-1))= Source ⇒

⇒ V(I,J)[CNWFN(J)-CSWFS(J)+DE+DW+DN+DS-0.5CE-0.5CW]=
=V(I+1,J)[-0.5CE+DE]+V(I-1,J)[0.5CW+DW]+V(I,J+1)[DN-CN
(1 - WFN(J))]+(I,J-1)[DS+CS(1-WFS(J))]+Source

and finally there is the requirement for the production of the discretization equation:

$$AE=DE-0.5CE$$
$$AW=DW+0.5CW$$
$$AN=DEN-CN(1-WFN(J))$$
$$AS=DS+CS(1-WFS(J))$$
$$AP=CNWFN(J)-CS*WFS(J)+DE+DW+DN+DS-0.5CE-0.5CW$$

Calculation of the pressure P (CALCP)

There is no differential equation for calculating the pressure. For this reason, a special technique is applied. The momentum equations, the source terms, also contain the pressure terms of the formula $(-\partial P/\partial x_i)$.

When these terms were expressed with the help of the finite differences, terms of the formula $(P_a - P_b)$ appeared in the calculations, which cannot be calculated, since the pressure is not known. In general, the momentum equations require the knowledge of the pressure distribution. So, we assume that the pressure distribution is known and we symbolize it by P*. It is declared in the subroutine INIT and is set equal to zero for each point of the grid -P. Then, however, u and v that are calculated are incorrect since the terms $(\partial P/\partial x_i)$ are calculated incorrectly. At this point the continuity equation comes to our aid, which has no pressure terms, but only speed terms. So, we should somehow correct the speeds u, v so that they verify the continuity equation. This is achieved in the following way:

If we assume that the corrections of u, v, P, are symbolized as u′, v′, P′. Then, it is valid:

$$P = P^* + P'$$
$$u = u^* + u'$$
$$v = v^* + v' \qquad \text{Actual values of P, u, v.}$$

In fact, all the previous calculations refer to the magnitudes p*, u*, v*.

All the momentum equations can be presented in the following general form:

$$\alpha_e u_e^* = \Sigma_i \alpha_{ie} u_i^* + b + \left(P_P^* - P_E^* \right) A_e \tag{6.1.10}$$

This equation, if written for the actual values of the speeds and pressures, will take the form:

$$\alpha_e u_e = \Sigma_i \alpha_{ie} u_i + b + \left(P_P - P_E \right) A_e \tag{6.1.11}$$

If we do the operation (6.1.11) – (6.1.10), we will get:

$$\alpha_e u'_e = \Sigma_i \alpha_{ie} u'_i + \left(P'_P - P'_E \right) A_e$$

The term $\Sigma_i \alpha_{ie} u'_i$, after many repetitions, will become very small, since eventually the speed corrections will be close to zero, therefore we can neglect it. Then, we will have:

$$\alpha_e u'_e = \left(P'_P - P'_E \right) A_e$$

To each point P two grids of type U and V correspond, as shown in the following figure. Thus, with the help of this relation we can calculate the corrections of all the speeds of the grid P.
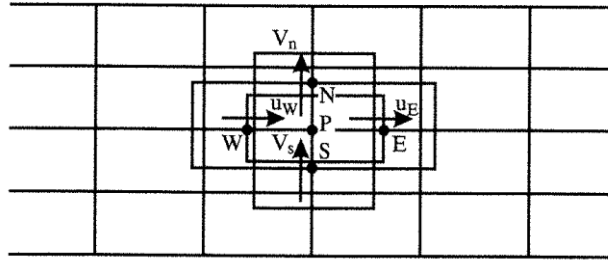
Figure 6.1.4 Grids of type U, V at the point P.

$$\alpha_n V'_n = -\left(P'_N - P'_P \right) A_N \rightarrow V'_n = -A_N/\alpha_n \left(P'_N - P'_P \right)$$

$$\alpha_s V'_s = -\left(P'_P - P'_S \right) A_S \rightarrow V'_s = -A_S/\alpha_s \left(P'_P - P'_S \right)$$

$$\alpha_w u'_w = -\left(P'_P - P'_W \right) A_w \rightarrow u'_W = -A_w/\alpha_w \left(P'_P - P'_W \right)$$

$$\alpha_e u'_e = -\left(P'_E - P'_P \right) A_E \rightarrow u'_e = -A_E/\alpha_e \left(P'_E - P'_P \right)$$

The terms A_i/α_i have been calculated in CALCU and CALCV. In particular, in each calculation of the term $(\partial P/\partial x_i)$ we had also calculated an area term (A_i) that we had named DU for CALCU and DV for CALCV. At the end of the subroutines the transformation takes place:

DU = DU/AP,   DV = DV/AP

The terms AP are α_i of the relations so they are written:

$$V'_n = DV(I, J + 1) (P'_P - P'_N)$$

$$V'_s = DV(I, J) (P'_S - P'_P)$$

$$u'_w = DU(I, J) (P'_W - P'_P)$$

$$u'_e = DU(I + 1, J) (P'_P - P'_E)$$

The terms DU and DV are defined in the grids u and v, respectively, so the coordinates I, J, correspond to these grids. Setting indicators also in the pressures, based of course on the –P grid and symbolizing the corrections P according to the program as PP we will have:

$$V'_n = DV(I, J + 1) \, (PP(I, J) - PP(I, J + 1)) \tag{6.1.12}$$

$$V'_s = DV(I, J) \, (PP(I, J - 1) - PP(I, J)) \tag{6.1.13}$$

$$u'_w = DU(I, J(PP(I - 1, J) - PP(I, J)) \tag{6.1.14}$$

$$u'_e = DU(I + 1, J)(PP(I, J) - PP(I + 1, J)) \tag{6.1.15}$$

In this way we managed to connect the pressure corrections with the speed corrections. To also calculate the speed corrections, we would need 2 equations. It is therefore advantageous to simply calculate the pressure corrections and through them to calculate the speed corrections with the help of the relations (6.1.12) to (6.1.15). It remains, therefore, to find a relation that connects the speeds between them. This relation will be given to us by the continuity equation. We write this equation once again:

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(r\varrho V) + \frac{\partial}{\partial x}(r\varrho u)\right] = 0 \tag{6.1.16}$$

Now we complete it in the control volume of P and we will have:

$$\iiint_{V_P} \frac{1}{r}\left[\frac{\partial}{\partial r}(r\varrho \, V) + \frac{\partial}{\partial x}(r\varrho u)\right] r d\varphi \, dx \, dr = 0 \Rightarrow$$

$$\iint_{A_P}\left[\frac{\partial}{\partial r}(r\varrho V) + \frac{\partial}{\partial x}(r\varrho u)\right] dx \, dr = 0 \Rightarrow$$

$$\iint \frac{\partial}{\partial r}(r\varrho V) \, dx \, dr + \iint \frac{\partial}{\partial x}(r\varrho u) \, dx \, dr = 0 \Rightarrow$$

$$\int_w^e (r\varrho V)\rceil_s^n \, dx + \int_s^n (r\varrho u)\rceil_w^e \, dr \tag{6.1.17}$$

We will now analyze each term separately, having previously placed $u = u^* + u'$ and $v = v^* + v'$.

$$\underbrace{\int_w^e (r\varrho v')\rceil_s^n \, dx + \int_s^n (r\varrho u')\rceil_w^e}_{(I)} = -\underbrace{\left[\int_w^e (r\varrho v^*)\rceil_s^n \, dx + \int_s^n (r\varrho u^*)\rceil_w^e \, dr\right]}_{(II)} \tag{6.1.18}$$

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Section (II) is a term which can be characterized as the source of the error of the correction calculations. Section (I) gives us the corrections. Because the only things that change in the previous relation are u, v, we will make a more general calculation with the help of the equation (6.1.17). This term is written as follows:

$$\int_w^e (r\varrho V)_n \, dx - \int_w^e (r\varrho V)_s \, dx$$

and based on what we have done so far, it is converted into FDE as follows:

$$\left[\frac{1}{2} (\varrho_N + \varrho_P) \, r_n \, V_n \, \Delta x_{ew} \right] - \left[r_s \frac{1}{2} (\varrho_P + \varrho_S) \, V_s \, \Delta x_{ew} \right] =$$

$$= \left[0.5 \, (\underbrace{DEN(I, J + 1) + DEN(I, J))}_{DENN} \, \underbrace{RV(J + 1) \, SEW(I)}_{AREAN} \, V_n \right] -$$

$$- \left[0.5 (\underbrace{DEN(I, J) + DEN(I, J - 1))}_{DENS} \, \underbrace{RV(J) SEW(I)}_{AREAS} \, V_s \right] =$$

$$= \underbrace{DENN \quad AREAN}_{Constant} \, V_n - \underbrace{DENS \quad AREAS}_{Constant} \, V_s$$

Constant   Constant   (6.1.19)

The second term is written as follows:

$$= \int_s^n (r\varrho u)_e \, dr - \int_s^n (r\varrho u)_w \, dr$$

and based on the analysis we have determined we will have:

$$\left[\frac{1}{2} (\varrho_P + \varrho_E) \, r_e \, u_e \, \Delta r_{ns} \right] - \left[\frac{1}{2} (\varrho_W + \varrho_P) \, r_w \, u_w \, \Delta r_{ns} \right] =$$

$$= \left[0.5 \, (\underbrace{DEN(I, J) + DEN(I + 1, J)}_{DENE} \, \underbrace{RCV(J) \, u_e \, SNS(J)}_{AREAEW} \right] -$$

$$- \left[0.5 (\underbrace{DEN(I, J) + DEN(I - 1, J))}_{DENW} \, \underbrace{RCV(J) \, u_w \, SNS(J)}_{AREAEW} \right] =$$

$$= DENE \quad AREAEW \, u_e - DENW \quad AREAEW \, u_w \qquad (6.1.20)$$

We can now determine the source term and the corrective section:

Source term:

From the equation (6.1.19), setting the speed V of the grid –V, we will get:

$$= \text{DENN AREAN } V(I,J+1) - \text{DENS AREAS } V(I,J-1) = CN-CS \qquad (6.1.21)$$

We do the same for the equation (6.1.20) and we have:

$$= \text{DENE AREAEW } U(I+1,J) - \text{DENW AREAEW } U(I,J) = CE - CW \qquad (6.1.22)$$

Correction term:

In (6.1.18) we set the terms of the corrective speeds as calculated in the equations (6.1.12-6.1.15) and we get:

$$= \underbrace{\text{DENN AREAN } DV\,(I,\,J+1)}_{AN(I,J)} (PP(I,J) - PP(I+1,\,J)) -$$

$$- \underbrace{\text{DENS AREAS } DV(I,J)}_{AS(I,J)} (PP(I,J-1) - PP(I,J)) =$$

$$= AN(I,J)\,(PP(I,J) - PP(I+1,\,J)) - AS(I,J)\,(PP(I,J-1) - PP(I,J))$$

In the equation (6.1.20) the corrective terms of the speeds are replaced and it follows:

$$= \underbrace{\text{DENE AREAEW } DU(I+1,J)}_{AE(I,J)} (PP(I,J) - PP(I+1,J)) -$$

$$- \underbrace{\text{DENW AREAW } DU(I,J)}_{AW(I,J)} (PP(I-1,J) - PP(I,J)) =$$

$$= AE(I,J)\,(PP(I,J) - PP(I+1,J)) - AW(I,J)\,(PP(I-1,J) - PP(I,J))$$

Thus, if we compose all these terms, we will have:

$$AP(I,J)\,PP(I,J) = AN(I,J)\,PP(I+1,J) + AS(I,J)\,PP(I,J-1) +$$
$$+ AW(I,J)\,PP(I-1,J) + AE(I,J)\,PP(I+1,J) - (CN - CS + CE - CW)$$

where:

$$AP(I,J) = AN(I,J) + AS(I,J) + AW(I,J) + AE(I,J)$$

Thus, we came up with a form similar to the one in which we have brought the other equations, so that it can be solved by the TDMA method, in the same subroutine (LISOLV). After the subroutine CALCP calculates the corrections PP(I,J), it finally corrects the speed values. According to (6.1.12-6.1.13) for the speed V, we will write:

$$V(I,J) = V(I,J) + \underbrace{DV(I,J)\ (PP(I,J-1) - PP(I,J))}$$
$$\quad\ \ \underset{V*}{|} \qquad\qquad\qquad \underset{V'}{}$$

According to (6.1.14-6.1.15) the speed V will be equal to:

$$U(I,J) = U(I,J) + \underbrace{DU(I,J)\ (PP(I-1,J) - PP(I,J))}$$
$$\quad\ \ \underset{U*}{|} \qquad\qquad\qquad \underset{U'}{}$$

The pressure will be calculated with the help of the corrective factor PP(I,J) not linearly, but with the help of the under-relaxation coefficient URFP. To do this, we also need a reference pressure. This is called PPREF and according to the program it expresses the pressure correction at the point (IPREF, JPREF) = (NJM1,2) where NJM1 = NJ-1 with NJ the end of the y-direction of the calculation grid. In this way the correct (corrected) pressure will be equal to:

$$P(I,J) = P\ (I,J) + URFP\ (PP(I,J) - PPREF)$$
$$\ \underset{P*}{|} \qquad\qquad\qquad\quad \underset{P'}{/} \qquad \underset{P'(IPREF\ ,\ JPREF)}{|}$$

Once this is done, the corrections PP(I,J) become zero and the program returns to its main part.

Calculation of the speed W (CALCW):

We will now write the momentum equation in the direction φ, where the speed is w:

$$\text{div}\ (\varrho\ \vec{u}\ w) - \text{div}\ (\mu\ \text{grad}\ w) = S_i$$

In cylindrical coordinates this becomes:

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\ (r\varrho\ Vw) + \frac{\partial}{\partial x}\ (r\varrho\ uw)\right] - \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\mu\ \frac{\partial w}{\partial r}\right) + \frac{\partial}{\partial x}\left(r\mu\ \frac{\partial w}{\partial x}\right)\right] = S_i$$
$$\qquad\qquad\qquad (1) \qquad\qquad\qquad\qquad\qquad\qquad\qquad (2)$$

(1) Convection terms

(2) Diffusion terms

Convection terms:

$$\iiint_{V_P} \frac{1}{r}\left(\frac{\partial}{\partial r}\ (r\varrho\ Vw)\right) dr\ dx\ rd\varphi = \iint_{A_P} \frac{\partial}{\partial r}\ (r\varrho\ Vw)\ dr\ dx$$

The calculation will be done on the grid of the point P. If the flow was 3D (net), then it would be calculated in a shifted grid in the φ-direction. The projection of this hypothetical grid on the plane gives us the P-grid:

$$\int_w^e (r\varrho\ Vw)\big]_s^n\ dx = \int_w^e (r\varrho\ Vw)_n\ dx - \int_w^e (r\varrho\ Vw)_s\ dx$$

<div align="center">(a)   (b)</div>

a) This term wants the density at the point n, where it is not stored. Thus, we get its average value between the points N, P, where it is stored:

$$\varrho\big]_n = \frac{\varrho_N - \varrho_P}{2} = 0.5\ (DEN(I, J+1) + DEN(I, J))$$

At n, the speed V is defined as V(I,J+1). Thus, it is valid:

$$GN = 0.5(DEN(I, J+1) + DEN(I, J))\ V(I, J+1)$$

w will be equal to:

$$w_n = \big[w_N + w_P\big]/2 = 0.5(W(I, J+1) + W(I, J))$$

The term

$$dx\big]_w^e = SEW(I)$$

and

$$r = RV(J+1)$$

since it concerns the speed V at n.

Thus, we have the term:

$$= GN\ w_n\ \underbrace{SEW(I)\ RV(J+1)}_{AREAN}$$

We set:

CN = GN AREAN

and the integral takes the form:

$$\text{Convection } N = 0.5\ CN\ (W(I,J) + W(I,J+1))$$

b) For this term the same are repeated, but with the indicator J-1 in the place of J:

$$GS = 0.5(DEN(I,J) + DEN(I,J\text{-}1))\ V(I,J)$$

Surface:

AREAS = RV(J) SEW(I)

Speed:

$$\mathbf{w_s} = 0.5(\mathbf{W(I, J)} + \mathbf{W(I, J - 1)})$$

setting:

CS = GS AREAS

we will get the integral:

$$\text{Convection } S = 0.5\ CS\ (W(I,J) + W(I,J\text{-}1))$$

For the second convection term it is valid:

$$\iint \frac{\partial}{\partial x}\ (r\varrho\ uw)\ dx\ dr = \int_s^n (r\varrho\ uw)\rfloor_w^e\ dr =$$

$$= \int_s^n (r\varrho\ uw)\rfloor_e\ dr - \int_s^n (r\varrho\ uw)\rfloor_w\ dr$$

(a)   (b)

a) Here we will have:

$$(\varrho u)_e = 0.5(DEN(I, J) + DEN(I + 1, J))U(I + 1, J) = GE$$

$$w_e = (0.5)(W_P + W_E) = 0.5(W(I, J) + W(I + 1, J))$$

$$r_e = RCV(J)$$

and:

$$dr\rfloor_s^n = SNS(I)$$

Thus, setting:

$$CE = \underbrace{CE\ RCV(I)\ SNS(J)}_{AREAEW}$$

$$\text{Convection } E = 0.5\ CE\ (W(I,J) + W(I+1,J))$$

b) Similarly, we will also have here:

$$(\varrho u)_w = 0.5(DEN(I, J) + DEN(I - 1, J))\ V(I,J) = GW$$

$$w_w = 0.5\ (W_P + W_W) = 0.5(W(I, J) + W(I - 1, J))$$

$$r_w = RCV(I)$$

and:

$$dr\rfloor_s = SNS(J)$$

Thus, setting:

CW = GW AREAEW

we will get:

Convection W = 0.5 CW (W(I,J) + W(I-1,J))

Diffusion terms:

$$\iint \frac{\partial}{\partial r}\left(r\mu\,\frac{\partial w}{\partial r}\right) dx\ dr = \int_w^e \left(r\mu\,\frac{\partial w}{\partial r}\right)\Big|_s^n dx =$$

$$\int_w^e \left(r\mu\,\frac{\partial w}{\partial r}\right)_n dx - \int_w^e \left(r\mu\,\frac{\partial w}{\partial r}\right)_s dx$$

       (a)   (b)

a) Here the terms will be equal to:

$$\left(\frac{\partial w}{\partial r}\right)_n = \frac{W_N + W_P}{\Delta r_{NP}} = \frac{W(I,J + 1) - W(I,J)}{DYNP(J)}$$

$$\mu_n = VISN = 0.5(VIS(I,J + 1) + VIS(I,J))$$

$$r_n = RV(J + 1)$$

and:

$$dx\rfloor_w^e = SEW(I)$$

therefore it is valid:

$$AREAN = RV(J+1)\ SEW(I)$$

We set:

$$DN = \frac{VISN \cdot AREAN}{DYNP(J)}$$

thus the integral is written:

Diffusion N = DN (W(I,J+1) - W(I,J))

b) Here, similarly, the terms are equal to:

$$\left(\frac{\partial w}{\partial r}\right)_s = \frac{w_P - w_S}{\Delta r_{PS}} = \frac{W(I,J) - W(I,J-1)}{DYPS(J)}$$

$$r_s = RV(J)$$

and:

$$dx\rfloor_w^e = SEW(I)$$

therefore it is valid:

AREAS = RV(J) SEW(I)

if we set:

$$\mu_s = VISS = 0.5(VIS(I,J) + VIS(I,J-1))$$

and

$$DS = \frac{VISS \cdot AREAS}{DYPS(J)}$$

we will get for the integral:

Diffusion S = DS (W(I,J) - W(I,J-1))

For the second diffusion term it is valid:

$$\iint \frac{\partial}{\partial x}\left(r\mu \frac{\partial w}{\partial x}\right) dx\, dr = \int_s^n \left(r\mu \frac{\partial w}{\partial x}\right)\Bigg|_e^w dr =$$

$$= \int_s^n \left(r\mu \frac{\partial w}{\partial x}\right)_w dr - \int_s^n \left(r\mu \frac{\partial w}{\partial x}\right)_e dr$$

a) These terms take the form:

$$\left(\frac{\partial w}{\partial x}\right)_w = \frac{w_P - w_W}{\Delta x_{PW}} = \frac{W(I,J) - W(I-1,J)}{DXPW(I)}$$

$$r_w = RCV(J)$$

$$dr\rfloor_s^n = SNS(J)$$

AREAEW = RCV(J) SNS(J)

$$\mu_w = VISW = 0.5(VIS(I,J) + VIS(I-1,J))$$

We set:

$$DW \doteq \frac{VISW \cdot AREAEW}{DXPW(I)}$$

and the integral becomes:

$$\text{Diffusion } W = DW \, (W(I,J) - W(I\text{-}1,J))$$

b) Similarly we will also have here:

$$\left(\frac{\partial w}{\partial x}\right)_e = \frac{w_E - w_P}{\Delta x_{PE}} = \frac{W(I+1,J) - W(I,J)}{DXEP(I)}$$

$$r_e = r_w = RCV(J)$$

and:

$$dr \big\rfloor_s^n = SNS(J)$$

$$AREAEN = RCV(J) \, SNS(J)$$

$$\mu_e = VISE = 0.5(VIS(I,J) + VIS(I+1,J))$$

and setting:

$$DE = \frac{VISE \cdot AREAEW}{DXEP(I)}$$

the integral will become:

$$\text{Diffusion } E = DE(W(I+1,J) - W(I,J))$$

The **source term** for this equation will be equal to:

$$\text{Source} = \frac{\mu}{r}\frac{\partial w}{\partial r} - \frac{1}{r^2}\frac{\partial}{\partial r}(r\mu \, w) - \frac{\varrho V w}{r}$$

This form can be simplified as follows:

$$\text{Source} = \frac{\mu}{r}\frac{\partial w}{\partial r} - \frac{r}{r^2}\frac{\partial}{\partial r}(\mu w) - \frac{1}{r^2}\mu w - \frac{\varrho V w}{r} =$$

$$= \frac{\mu}{r}\frac{\partial w}{\partial r} - \frac{1}{r}\frac{\partial}{\partial r}(\mu w) - \frac{\mu w}{r^2} - \frac{\varrho V w}{r} =$$

$$= \frac{\mu}{r}\frac{\partial w}{\partial r} - \left(\frac{w}{r}\frac{\partial \mu}{\partial r} + \frac{\mu}{r}\frac{\partial w}{\partial r}\right) - \frac{\mu w}{r^2} - \frac{\varrho V w}{r} =$$

$$= \frac{w}{r}\frac{\partial \mu}{\partial r} - \frac{\mu w}{r^2} - \frac{\varrho V w}{r} =$$

$$- w\left[\frac{1}{r}\frac{\partial \mu}{\partial r} + \frac{\mu}{r^2} + \frac{\varrho V}{r}\right]$$

With the same handling, as for the u, v-speeds, we will also have here:

$$w_p = W(I,J)$$

$$\frac{1}{r}\frac{\partial \mu}{\partial r}\Big|_P = \frac{1}{r_P}\frac{\mu|_n - \mu|_s}{\Delta r_{ns}} = \frac{VISN - VISS}{SNS(J)}\frac{1}{RCV(J)}$$

$$\frac{\mu}{r^2}\Big|_P = \frac{VIS(I, J)}{(RCV(J))^2}$$

This term does not exist in the Cartesian coordinate system (INDCOS = 1).

$$\frac{\varrho V}{r}\Big|_P = \frac{DEN(I,J)\,V|_P}{RCV(J)} = \frac{DEN(I,J)\,0.5(V(I,J) + V(I,J+1))}{RCV(J)}$$

and:

$$VOL = RCV(J)\,SNS(J)\,SEW(I)$$

Adding these terms, we calculate the source. Then, we set:

Source = (Sum) X VOL

Thus, finally, the following will apply to the calculation of the w-speed:

[Convection N - Convection S] + [Convection E - Convection W] -
- [Diffusion N - Diffusion S] - [Diffusion E - Diffusion W] = Source

This expression eventually becomes:

0.5CN (W(I,J) + W(I,J+1)) - 0.5CS (W(I,J) + W(I,J-1)) +
+ 0.5CE (W(I,J) + W(I+1,J)) - 0.5CW (W(I,J) + W(I-1,J)) -
- DN (W(I,J+1) - W(I,J)) + DS (W(I,J) - W(I,J-1)) + DW (W(I,J) -
- W(I-1,J)) - DE (W(I+1,J) - W(I,J)) = Source

$\Rightarrow$ W(I,J) [0.5CN + 0.5CE - 0.5CS - 0.5CW + DN + DS + DW + DE] =
= W(I+1,J) [DE - 0.5CE] + W(I-1,J) [DW + 0.5CW] +
+ W(I,J+1) [DN - 0.5CN] + W(I,J-1) [DS + 0.5CS] + Source

Finally, we set the known coefficients:

AE = DE - 0.5CE

AW = DW + 0.5CW

AN = DN - 0.5CN

AS = DS + 0.5CS

AP = 0.5 (CN + CE - CS - CW) + DN + DS + DW + DE

Turbulence model (CALCTE, CALCED)

The turbulent flows are non-constant and a complete description of them would require resolving the relevant equations. But because the vortex scale is about 10^{-3} sec, the calculations would be anti-economic. A practical solution is to convert the governing equations into equations of average time values, model the occurring turbulent terms and solve them. The model used in TEACH is that of the two equations k-ε, where k the turbulent kinetic energy and ε the scattering rate (degeneration) of the turbulent kinetic energy. The active viscosity is obtained from the relation:

$$\mu_{eff} = \frac{C_\mu \varrho k^2}{\varepsilon}$$ (6.1.23)

The equations of these two variables are as follows in general form:

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(\varrho u r \Phi) + \frac{\partial}{\partial r}(\varrho r v \Phi) - \frac{\partial}{\partial x}\left(r\Gamma_{eff,\Phi}\frac{\partial\Phi}{\partial x}\right) - \frac{\partial}{\partial r}\left(r\Gamma_{eff,\Phi}\frac{\partial\Phi}{\partial r}\right)\right] = S_\Phi$$

where Φ becomes k and ε, while the other coefficients are equal to:

$$\Gamma_{eff,\Phi} = \mu_{eff}/\sigma_{eff,\Phi} = \mu/Pr$$

The source term is, respectively:

$$S_\varkappa = G - C_D \varrho\varepsilon$$

and:

$$S_\varepsilon = C_1 G\varepsilon/k - C_2 \varrho\varepsilon^2/k$$

where the common coefficient G is equal to:

$$G = \mu \left[\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial r} - \frac{w}{r} \right)^2 \right] + \mu \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 \left(\frac{\partial v}{\partial r} \right)^2 \right] + \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right)^2 \right\} + \mu \left(\frac{v}{r} \right)^2$$

The last term exists only in the polar cylindrical coordinate system (where INDCOS = 2). The values of C_i, $\sigma_{eff,\Phi}$, are calculated basically experimentally and in TEACH they have the following values:

Magnitude	Symbol
$C\mu = 0.09$	CMU
$C_D = 1.0$	CD
$C_1 = 1.44$	C1
$C_2 = 1.92$	C2
$\sigma_{eff,k} = 1.0$	PRTE
$\sigma_{eff,e} = 1.21$	PRED

k is symbolized as TE(I,J) and ε as ED(I,J). The calculation grid is P. The integrations are executed with respect to the general variable Φ, since the equations of k, ε are the same. The convection terms of the differential equation are:

$$\frac{1}{r} \left[\frac{\partial}{\partial x} (\varrho u r \Phi) + \frac{\partial}{\partial r} (\varrho r v \Phi) \right]$$

We analyze each one separately:

$$\iiint_{V_P} \frac{1}{r} \frac{\partial}{\partial x} (\varrho u r \Phi) \, r d\varphi \, dx \, dy = \iint_{A_P} \frac{\partial}{\partial x} (\varrho u r \Phi) \, dx \, dy =$$

$$\int_s^n (\varrho u r \Phi) \Big|_w^e \, dr = \int_s^n (\varrho u r \Phi)_e \, dr - \int_s^n (\varrho u r \Phi)_w \, dr$$

(a)   (b)

a) We have calculated terms of this form once again, so we proceed without detailed explanations.

$(\varrho u)_e = (\varrho_P + \varrho_E)/2 \; U(I + I,J) = 0.5(DEN(I,J) + DEN(I + 1,J)) \; U(I + 1,J) = GE$

$\Phi_e = (\Phi_P + \Phi_E)/2 = 0.5(\Phi(I,J) + \Phi(I + 1,J))$

$r_e = RCV(J)$

$dr \Big|_w^e = SNS(J)$

so we set as a surface:

AREAEW = RCV(J) SNS(J)

Thus, setting

CE = GE AREAEW

the integral will be written as:

$$\text{Convection E} = \text{CE } 0.5(\Phi(I,J) + \Phi(I+1,J))$$

b) Similarly, for this term it will also be valid:

$$(\varrho u)_w = (\varrho_P + \varrho_W)/2 \, U(I,J) = 0.5(DEN(I,J) + DEN(I-1,J)) \, U(I,J) = GW$$

$$r_w = r_e = RCV(J)$$

and:

$$dr \rfloor_s^n = SNS(J)$$

therefore we have again AREAEW and still:

$$\Phi_w = (\Phi_P + \Phi_W)/2 = 0.5(\Phi(I,J) + \Phi(I-1,J))$$

Then, CW = GW AREAEW and the integral takes the form:

$$\text{Convection W} = 0.5CW(\Phi(I,J) + \Phi(I-1,J))$$

b) The next convection term is equal to:

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(\varrho r v \Phi)\right]$$

which, when completed, will give:

$$\iint \frac{\partial}{\partial r}(\varrho r v \Phi) \, dr \, dx = \int_w^e (\varrho r v \Phi) \rfloor_s^n \, dx =$$

$$\int_w^e (\varrho r v \Phi)_n \, dx - \int_w^e (\varrho r v \Phi)_s \, dx$$

       (a)   (b)

a) The analysis of these terms is the same as before.

$$(\varrho v)_n = (\varrho_P + \varrho_N)/2 \, V(I,J) = 0.5(DEN(I,J) + DEN(I,J+1)) \, V(I,J+1) = GN$$

$$r_n = RV(J+1)$$

and:

$$dx \rfloor_w^e = \text{SEW(I)}$$

therefore the surface is equal to:

$$\text{AREAN} = \text{RV(J+1) SEW(I)}$$

Finally, we set CN = GN AREAN and thus the integral takes the form:

$$\Phi_n = (\Phi_P + \Phi_N)/2 = 0.5(\Phi(I,J) + \Phi(I,J+1))$$

$$\text{Convection N} = 0.5\text{CN}(\Phi(I,J) + \Phi(I,J+1))$$

b) Similarly, it will also be valid here:

$$(\varrho v)_S = (\varrho_P + \varrho_S)/2 \; V(I,J) = 0.5(\text{DEN}(I,J) + \text{DEN}(I,J-1)) \; V(I,J) = \text{GS}$$

$$r_S = \text{RV(J)}$$

and:

$$dx \rfloor_w^e = \text{SEW(I)}$$

so the surface is equal to:

$$\text{AREAS} = \text{RV(J)*SEW(I)}$$

$$\Phi_S = (\Phi_P + \Phi_S)/2 = 0.5(\Phi(I,J) + \Phi(I,J-1))$$

We set CS = GS AREAS and thus the integral is written as follows:

$$\text{Convection S} = 0.5\text{CS}(\Phi(I,J) + \Phi(I,J+1))$$

The diffusion terms are:

$$-\frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\Gamma_{\text{eff},\Phi}\frac{\partial\Phi}{\partial x}\right) - \frac{\partial}{\partial r}\left(r\Gamma_{\text{eff},\Phi}\frac{\partial\Phi}{\partial r}\right)\right]$$

$\Gamma_{\text{eff}} = \mu_{\text{eff}}/\sigma_{\text{eff},\Phi} = \mu/\text{Pr}$, where PR becomes for $\Phi = k \rightarrow$ PRTE and for $\Phi = \varepsilon \rightarrow$ PRED.

We analyze each term separately:

$$\iint_{A_P} \frac{\partial}{\partial x}\left(r\frac{\mu}{\text{Pr}}\frac{\partial\Phi}{\partial x}\right)dx\,dr = \int_s^n r\frac{\mu}{\text{Pr}}\frac{\partial\Phi}{\partial x}\rfloor_w^e\,dr =$$

$$= \int_s^n \left(r\frac{\mu}{\text{Pr}}\frac{\partial\Phi}{\partial x}\right)_e dr - \int_s^n \left(r\frac{\mu}{\text{Pr}}\frac{\partial\Phi}{\partial x}\right)_w dr$$

<div align="center">(a)   (b)</div>

The term μ/Pr is complex, since it contains μ which changes at the ends of the control volume and Pr which is constant throughout the calculation space. Thus, the following will apply:

$$(\mu/Pr)_e = (\mu_P + \mu_E)/(2Pr) = 0.5(VIS(I,J) + VIS(I+1,J))/PR$$

$$r_e = RCV(J)$$

and:

$$dr\big|_s^n = SNS(J)$$

thus, the surface is equal to:

$$AREAEW = RCV(J)\ SNS(J)$$

$$\left(\frac{\partial\Phi}{\partial x}\right)_e = (\Phi_E - \Phi_P)/\Delta x_{EP} = (\Phi(I+1,J) - \Phi(I,J))/DXEP(I)$$

We set:

$$GAME = (\mu/Pr)_e$$

and:

$$DE = GAME\ AREAEW/DXEP(I)$$

Thus, the integral takes the form:

$$\text{Diffusion E} = DE\ (\Phi(I+1,J) - \Phi(I,J))$$

b) Similarly for this term it will also be valid:

$$(\mu/Pr)_w = GAMW = (\mu_w + \mu_P)/(2Pr) = 0.5(VIS(I,J) + VIS(I-1,J))/PR$$

$$r_w = r_e = RCV(J)$$

and:

$$dr\big|_s^n = SNS(J)$$

so the surface becomes $AREAEW = RCV(J)\ SNS(J)$

$$\left(\frac{\partial\Phi}{\partial x}\right)_w = (\Phi_P - \Phi_W)/\Delta x_{PW} = (\Phi(I,J) - \Phi(I-1,J))/DXPW(I)$$

Thus, we set:

$$DW = GAMW\ AREAEW/DXPW(I)$$

so the integral will be:

Diffusion W = DW (Φ(I,J) - Φ(I-1,J))

$$\iint_{A_P} \frac{\partial}{\partial r}\left(r \frac{\mu}{Pr} \frac{\partial \Phi}{\partial r} \right) dx\, dr = \int_w^e \left(r \frac{\mu}{Pr} \frac{\partial \Phi}{\partial r} \right)\Big|_s^n dx =$$

$$\int_w^e \left(r \frac{\mu}{Pr} \frac{\partial \Phi}{\partial r} \right)_n dx - \int_w^e \left(r \frac{\mu}{Pr} \frac{\partial \Phi}{\partial r} \right)_s dx$$

a) Similarly to the first term it will be valid:

$$(\mu/Pr)_n = (\mu_P + \mu_N)/(2Pr) = 0.5(VIS(I,J) + VIS(I,J+1))/PR = GAMN$$

$$r_n = RV(J+1)$$

and:

$$dx\Big\rfloor_w^e = SEW(I)$$

so we have the surface AREAN = RV(J+1) SEW(I)

$$\left(\frac{\partial \Phi}{\partial r} \right)_n = (\Phi_N - \Phi_P)/\Delta r_{PN} = (\Phi(I,J+1) - \Phi(I,J))/DYNP(J)$$

We set:

$$DN = GAMN\ AREAN/DYNP(I)$$

therefore the integral will be:

Diffusion N = DN (Φ(I,J+1)-Φ(I,J))

b) Similarly it will also be valid here:

$$(\mu/Pr)_s = (\mu_P + \mu_S)/(2Pr) = GAMS = 0.5(VIS(I,J) + VIS(I,J-1))/PR$$

$$r_s\, dx\Big\rfloor_w^e = AREAS = RV(I)\ SEW(I)$$

$$\left(\frac{\partial \Phi}{\partial r} \right)_s = (\Phi_P - \Phi_S)/\Delta r_{PS} = (\Phi(I,J) - \Phi(I,J-1))/DYPS(I)$$

We set:

$$DS = GAMS\ AREAS/DYPS(J)$$

therefore the integral will be:

Diffusion S = DS (Φ(I,J)-Φ(I,J-1))

Source Terms:

In the source terms the basic parameter is G. We will calculate each term separately:

$$\left(\frac{\partial w}{\partial x}\right)_P = (w_e - w_w)/\Delta x_{ew} = \left[0.5\,(W_P + W_{E)}\right]/SEW(I) -$$

$$DWDX = [0.5(W(I,J) + W(I + 1,J)) - 0.5(W(I,J) + W(I - 1,J))]/SEW(I)$$

$$\left(\frac{\partial w}{\partial r}\right) = (w_n - w_s)/\Delta r_{ns} = [0.5(W_N + W_P) - 0.5(W_P + W_S)]/SNS(J) -$$

$$DWDY = [0.5(W(I,J + 1) + W(I,J) - 0.5(W(I,J) + W(I,J - 1))]/SNS(J)$$

$$(w/r)_P = WDR = W(I,J)/RCV(J)$$

$$\left(\frac{\partial u}{\partial x}\right)_P = DUDX = [U(I + 1, J) - U(I, J)]/SEW(I)$$

$$\left(\frac{\partial v}{\partial r}\right)_P = DVDY = [V(I,J + 1) - V(I,J)]/SNS(J)$$

$$\left(\frac{\partial u}{\partial r}\right)_P = (U_n - U_s)/\Delta r_{ns} = DUDY$$

$$\Delta r_{ns} = SNS(J)$$

The term U_n will be calculated as the average value of the values at the 4 neighboring points 1, 2, w, e (see figure 6.1.3), while U_s at the 4 neighboring points w, e, 3, 4. Thus, we have:

$$U_n = 1/4(U(I, J) + U(I, J + 1) + U(I + 1, J) + U(I + 1, J + 1))$$

$$U_s = 1/4(U(I,J) + U(I + 1,J) + U(I,J - 1) + U(I + 1,J - 1))$$

$$\left(\frac{\partial v}{\partial x}\right)_P = (V_e - V_w)/\Delta x_{ew} = DVDX$$

$$\Delta x_{ew} = SEW(I)$$

As before, the term V_e will be calculated as the average value of the speeds at the 4 neighboring points n, 3′, 4′, s. V_w will be calculated with the help of the neighboring points 1′, 2′, s, n. Thus, we have:

$$V_e = 1/4(V(I,J) + V(I,J + 1) + V(I + 1,J) + V(I + 1,J + 1))$$

$$V_w = 1/4(V(I,J) + V(I,J + 1) + V(I - 1,J) + V(I - 1,J + 1))$$

$$(v/r)_P = 0.5\,[(v/r)_n + (V/r)_s] = 0.5[V(I,J)/RV(J + 1) + V(I,J)/RV(J)]$$

The term V(I,J)/RV(J) is set as VDR, so this term is written as follows:

$$0.5[VDR+V(I, J+1)/RV(J+1)]$$

$\mu_P \rightarrow$ this term, because it is stored at P, always takes the value VIS(I,J). Finally, all these terms are added and multiplied depending on the volume:

$$VOL = RCV(J) \, SEW(I) \, SNS(J)$$

in order to give us the term G which in the program is calculated once in CALCTE with the name GEN(I,J). All the terms are composed as follows:

$$[\text{Convection N - Convection S}] + [\text{Convection E - Convection W}]+$$
$$[\text{Diffusion N - Diffusion S}] - [\text{Diffusion E - Diffusion W}] = \text{Source}$$

This expression is written as follows:

$$0.5CE(\Phi(I,J) + \Phi(I+1,J)) - 0.5CN(\Phi(I,J) + \Phi(I-1,J)) +$$
$$+ 0.5CN(\Phi(I,J) + \Phi(I,J+1)) - 0.5CS(\Phi(I,J) + \Phi(I,J-1)) -$$
$$- DE(\Phi(I+1,J) - \Phi(I,J)) + DW(\Phi(I,J) - \Phi(I-1,J)) -$$
$$- DN(\Phi(I,J+1) - \Phi(I,J)) + DS(\Phi(I,J) - \Phi(I,J-1)) = \text{Source} \rightarrow$$

$$\rightarrow \Phi(I,J)[0.5CE - 0.5CW + 0.5CN - 0.5CS + DE + DW + DS + DN] =$$
$$= \Phi(I+1,J) [DE - 0.5CE] + \Phi(I-1,J) [DW + 0.5CW] +$$
$$+ \Phi(I,J+1) [DN - 0.5CN] + \Phi(I,J-1) [DS + 0.5CS] + \text{Source.}$$

Finally, we set the terms:

$$AE = DE - 0.5CE$$

$$AW = DW + 0.5CW$$

$$AN = DN - 0.5CN$$

$$AS = DS + 0.5CS$$

$$AP = 0.5(CE - CW + CN - CS) + DE + DW + DS + DN$$

LISOLV is then called, to calculate TE(I,J) in CALCTE and ED(I,J) in CALCED. Finally, the correction of μ through ε, k, according to the relation (6.1.23), remains. This is done in the subroutine PROPS which, when the logic indicator INPRO is true in each scan, is called upon to modify the values of the density, the viscosity and the flow characteristics. Under the heading VISCOSITY the following are done: the old value of the viscosity is stored in the variable VISOLD.

If the calculated value of ED(I,J) becomes zero, it will cause an error (run time error) since it is in the denominator in the relation (6.1.23). Therefore, a check is made for its value. If it is equal to zero, the viscosity takes the predetermined initial value VISCOS = 1,9 x 10^{-5}.

$$\mathrm{VIS(I,J)} = \underbrace{\mathrm{DEN(I,J)}\ \mathrm{TE(I,J)}^2\ \mathrm{CMU/ED(I,J)}} + \underbrace{\mathrm{VISCOS}}$$
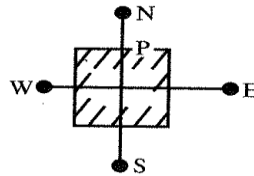
Turbulent viscosity   laminar flow viscosity

With the under-relaxation coefficient URFVIS the viscosity value finally becomes:

$$\mathrm{VIS(I,J)} = \mathrm{URFVIS\,VIS(I,J)} + (1 - \mathrm{URFVIS})\mathrm{VISOLD}$$

As we saw in our analysis, all the differential equations end up in a form, such as the following:

$$\alpha_P^\Phi\ \Phi_P = \Sigma_j \alpha_j^\Phi \Phi_j + S_u^\Phi$$

where the term Σ_j indicates the sum at the nodes N, S, E, W of the grid corresponding to the control volume in which we do the integration. In all the differential equations there was a source term which we symbolized by S_Φ and which we completed in the control volume in a different way than in the other parameters. In fact, we did linearization of the source in the general form.

$$S_\Phi = S_P^\Phi\ \Phi_P + S_u^\Phi$$

In the subroutines CALCx the terms S_p and S_u are symbolized as SP(I,J) and SU(I,J) respectively. The integration now takes the form:

$$\int_v S_\Phi\ dV = \left(S_P^\Phi\ \Phi_P + S_u^\Phi \right) V$$

The term S_p does not often appear and only S_u exists, while in other cases both appear.

The term S_p is contained in α_p^Φ, which is equal to:

$$\alpha_P^\Phi = \Sigma_j \alpha_j - S_p^\Phi\ V = \alpha_N^\Phi + \alpha_E^\Phi + \alpha_w^\Phi + \alpha_s^\Phi - S_P^\Phi\ V$$

In the program the terms α_j are the terms AN, AS etc. that we defined at the end of each integration.

In order to have uniformity on the line of the CALCx subroutines in cases where there is no term S_P^Φ, the program sets as S_P^Φ the term:

$$S_P^\Phi = -(\mathrm{CW} - \mathrm{CS} + \mathrm{CE} - \mathrm{CW})$$

while at the same time it sets as a source term of type S_u^Φ the term:

$$S_u^\Phi = (CN - CS + CE - CW) \, \Phi(I,J)$$

In this way, the two terms are cancelled algebraically, leaving the actual source terms. In any case, however, because the term S_P^Φ is subtracted from the sum of α_j, it is possible to have $\alpha_P < 0$. This should not be the case under any circumstances, as most physical processes do not have a source term dependent on the magnitude Φ with positive slope. This would lead to instability, since an increase in Φ_P would lead to an increase in the source term S_P^Φ, which of course leads to an increase in Φ_P, with the result that, if there is no mechanism to abduct Φ_P, unrealistic processes take place.

For this reason in each CALCx a check is made for S_P so that, if it has a positive value, it is zeroed. The linearization of the source terms presupposes the existence of an iterative process for the correction of the values. A better approach is achieved by under-relaxing the terms S_u^Φ by means of an under-relaxation coefficient. For this reason, in CALCx S_u^Φ is set as follows:

$$\alpha_P^\Phi = \alpha_P^\Phi / URFx$$

$$S_u^\Phi = S_u^\Phi + (1 - URFx)\alpha_P^\Phi \Phi_P$$

where URFx is the under-relaxation coefficient, which takes different values, depending on the case (URFU, URFV etc.). In every calculation process there are divergences, which, when they are small enough, give us the correct results. In the subroutines CALCx the divergences are defined by the characteristic RESORx and are:

$$RESORx = \Sigma_j \alpha_j^\Phi \Phi_j + S_u^\Phi - \alpha_P^\Phi \Phi_P$$

This variable is called remainder and is summed for all the grid points. Its reduction also shows us the convergence of the iterative calculation process.

On each side of the control volume we have a sum of quantities Φ, due to diffusion and convection. Let's take, for example, the end e of a control volume.
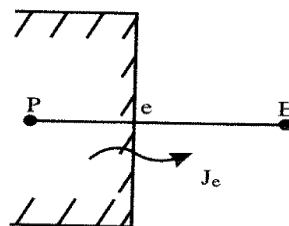
The calculation of the integrals at e always gives us a term of the form:

$$J_e = 0.5CE(\Phi(I+1,J) + \Phi(I,J) - DE(\Phi(I+1,J) - \Phi(I,J))$$

This term expresses the net reduced flow rate through the wall e due to diffusion and convection. We can write it as follows:

$$J_e = (0.5 \ (CE+DE) \ \underset{\Phi_P}{\underbrace{\Phi(I,J)}} - \underset{\Phi_E}{\underbrace{\Phi(I+1,J)}} [DE-0.5CE]$$

If we set:

$$A = DE + 0.5CE$$
$$B = DE - 0.5CE$$

we can write more simply:

$$J_e = A\Phi_P - B\Phi_E$$

If A and B are negative, then, when Φ_P increases, J_e decreases and, when Φ_E decreases, J_e also decreases. This result is wrong and arises, because we ignored the effect of u_e on Φ_E. It should, therefore, be valid:

$$A \geq 0$$
$$B \geq 0$$

which means:

$$DE + 0.5CE \geq 0$$
$$DE - 0.5CE \geq 0$$

If we assume that 0,5CE is positive, then we can write:

$$DE + |0.5CE| \geq 0 \tag{6.1.24}$$
$$DE - |0.5CE| \geq 0$$

For (6.1.24) to be valid, it is enough to set in the position of DE the maximum of DE, |0.5CE|. If we set DE, our basic relation arises, while, if we set |0.5CE|, we have (a) positive and (b) zero. If again |0.5CE| is negative, then it will be valid:

$$DE - |0.5CE| \geq 0 \tag{6.1.25}$$
$$DE + |0.5CE| \geq 0$$

which is the same as above. This is what the subroutines CALCx of TEACH do, where with commands of the formula:

$$(\text{CALCU})\longrightarrow$$

$$AE(I,J) = AMAX1(ABS(0.5CE), DE) - 0.5CE$$

they ensure that relations like that of (6.1.25) will be valid. In other cases, the comparison is more complex. From the calculations we made in CALCU we have, for example:

$$J_e = CE(U(I,J)WFE(I) + U(I+1,J)(1 - WFE(I)) - DE(U(I+1,J) - U(I,J)) \rightarrow$$

$$\rightarrow J_e = U(I,J)(DE + CE\ WFE(I)) - U(I+1,J)(DE - CE(1 - WFE(I)))$$

where, according to what we have said, the following should be valid:

$$DE + CE\ WFE(I) \geq 0$$

$$DE - CE(1 - WFE(I)) \geq 0 \tag{6.1.26}$$

The relation (6.1.26) is satisfied in the following way:

$$DE = AMAX1(DE, -WFE(I)\ CE, CE(1 - WFE(I)))$$

$$AE(I,J) = DE - (1 - WFE(I))\ CE$$

and finally we demand:

$$AE = DE - 0.5CE$$

$$AW = DW + 0.5CW$$

$$AN = DN - CN(1 - WFN(J))$$

$$AS = DS + CS(1 - WFS(J))$$

$$AP = CN\ WFN(J) - CS\ WFS(J) + DE + DW + DN + DS - 0.5CE - 0.5CW$$

CHAPTER 7

IMPLEMENTATION OF THE COMPUTATIONAL PROGRAM

7.1 Implementation of the Computational Program

The first step in implementing the computational program for a specific problem is to determine the resolution area and the grid.
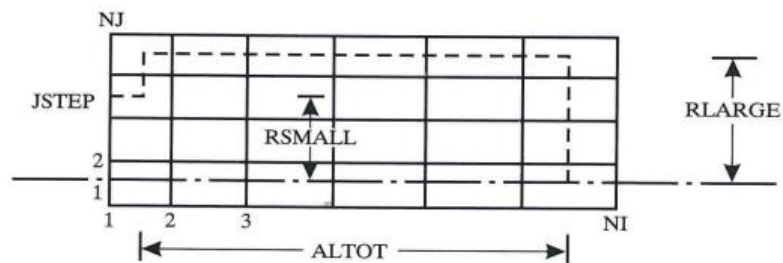
The resolution area is limited by the input plane, the cylindrical wall (outlines), the axis of symmetry and the output plane. The calculations start from the plane of STEP. The grid used may be uniform or gradually increasing with an expansion coefficient EPSX (1.2 or smaller), in order to allow the grid to be concentrated in the recirculation or near the wall.

The handling near the wall must be careful in introducing the limit conditions (subroutine PROMOD).

On the axis of symmetry the total vertical flow of the variables is equal to zero $(\partial\Phi/\partial r = 0)$. At the input, uniform variables are defined for all the variables:

$$U = U_{in}$$
$$V = 0$$
$$k = k_{in} = iU_{in}^2$$
$$\varepsilon = \varepsilon_{in} = k_{in}^{3/2}/L$$

where i = turbulence intensity coefficient and:

L = λ D/2   where λ = length scale factor

In the equation P´, however, the output speeds are required for the mass conservation. The speeds are obtained at the output with the assumption of zero slope in combination with the application of the total mass balance in the entire flow solution area.

In **Section 0** of the subroutine CONTRO, where the program is mainly controlled, DIMENSION, COMNON elements and, also, the scan numbers (NSWPΦ) of the repetition lines are declared. Another important parameter identified here is the maximum dimensions of the Φ-tables (IT, JT).

Section 1 defines the control indicators and the parameters related to the grid, the selection of the equations to be resolved, the determination of the values of the fluid properties, the turbulence constants, the limit values and the parameters for the control of the program and the printing. The term "grid determination" defines INDCOS, NI, NJ, the cylinder spoke RLARGE and the cylinder length of calculations ALTOT. Also, NI = NJ = 30 define a 30x30 grid (give 28x28) cells. RSMALL is the spoke of the small diameter of STEP. The grid coordinates X(1), Y(1) are defined as symmetric of X(2) and Y(2), respectively.

In *Section 1* the parameters INCALΦ are declared, to select the equations to be resolved (u, v, p´, k, ε and μ). The coordinates of the position (IPREF, JPREF) for the pressure determination, the printing parameters MAXIT and the under-relaxation coefficients URFΦ are given.

In **Section 2** of CONTRO the initial values (zero) are determined for the variables Φ and the geometric quantities are defined through the subroutine INIT. The initial fluid properties are also calculated through PROPS. The subroutine INLET is called and then the input values and the improved initial distributions of the variables are determined. The values within the flow field are equalized with the input values with the speed U and the mass flow rate (FLOWIN) is estimated. The initial information and the initial fields Φ are printed and the normalized quantities of the mass and the momentum sources are calculated. The mass sources are normalized with FLOWIN and the momentum sources with XMONIN(=FLOWINxUIN).

In **Section 3** of CONTRO the iterative "LOOP" of the achievement of "CALL" in the different subroutines Φ is arranged and care is taken for intermediate prints in multiple of the INDPRI repetitions. Also, the control of the termination of the calculation process based on SORCE takes place.

In **Section 4** the final values of the tables Φ are printed via the "CALL" command of the subroutine PRINT.

In **Section 1** of the subroutine INIT the calculations of the coordinates of the position of the speeds (XU(I), YV(J)), the dimensions of the cells (SEW(I), SNS(J), and so on) are done.

In **Section 2** the variables are given the initial zero values apart from DEN(I,J) = DENSIT and VIS(I,J) = VISCOS.

The subroutine PROPS calculates the fluid properties (here μ_{eff}). The exchange coefficients μ_{eff}/σ_k and $\mu_{eff}/\sigma_\varepsilon$ are calculated in CALCTE and CALCED respectively ($\sigma_k \rightarrow$ PRTE, $\sigma_\varepsilon \rightarrow$ PRED).

The structure and the indexing of the subroutine CALCU is examined to find details for all the subroutines CALCΦ. GN, GS etc. are the mass flows, CN, CS etc. and DN, DS etc. represent coefficients of transfer and diffusion, respectively. SMP represents the local mass production M_P and the "hybrid" difference method is executed through the functions AMAX1(). The complete source of the finite difference equation is assembled, since the subroutine MODU is called in PROMOD for the necessary source modifications at the limits etc. It should be noted that prior to LISOLV for the application of the LBL method, indirect under-relaxation is performed by modifying the finite difference equation coefficients.

The introduction of the limit conditions is done in PROMOD by modifying the appropriate coefficients. Consequence of the "upper wall" is the introduction of shear force along the cylindrical wall. The procedure followed is the selection of an appropriate shear equation in accordance with y^+, the calculation of τ_w (TAUN(I)) and its introduction by SP(I,J). Usually, the shear effect is removed, by setting AN(I,J) = 0.

The "west wall" and the "axis of symmetry" are treated by setting the "vertical flows" equal to zero. We handle the output speeds (U(NI,J)) with a zero degree view in combination with the mass balance. The handling of the V-momentum equation is similar to that of the U-momentum, except that the calculations are now made for the "west wall".

In the subroutine PROMOD modifications are made for the k-equation, setting the vertical flows on the wall equal to zero and adjusting the production term (GENCOU). The practice of the "zero slope" on the axis of symmetry with regulation of GEN(I,2) is used.

ε is determined at the adjacent nodes of the wall by the source linearization method (GREAT $=10^{30}$ in CONTRO) and on the axis of symmetry the vertical flow is set equal to zero.

The subroutine PROMOD plays a very important role in the program, because there the sources and the limit conditions are modified, in order to adapt to the particular problem. It is divided into Sections, each of which has a

special variable Φ, starting from ENTRY and ending in RETURN. The points of the beginnings have been named MODΦ and Φ are the relevant variables (U, V, P and so on).

The subroutine LISOLV performs the repetitions of LBL, scanning in the direction W-E. The coefficients of the iterative relation are assembled along the line N-S and the iterative relation is then used to calculate Φ.

The tables of the variables, the coordinates and the corresponding indicators are printed by the subroutine PRINT, as well as the names of the tables. The tables of the variables PHI(I,J) are printed for the entire field from (ISTART, JSTART) to (NI, NS) of the coordinates X(I), Y(J), as well as the labels (HEAD) of each table (PHI(I,J)).

ΣΥΜΠΕΡΑΣΜΑΤΑ

Στο πλαίσιο της παρούσας διπλωματικής εργασίας μελετήσαμε το θέμα της ρευστομηχανικής και πιο συγκεκριμένα το πρόγραμμα προσομοίωσης teach-t. Αυτό μας βοήθησε να εντρυφήσουμε στους τεχνικούς όρους της ειδικότητας στην αγγλική γλώσσα και τον τρόπο μετάφρασης ενός εγχειριδίου από την ελληνική στην αγγλική γλώσσα. Η όλη διαδικασία αποτέλεσε μια εποικοδομητική ενασχόληση, τόσο με το αντικείμενο της υπολογιστικής ρευστομηχανικής, όσο και με την καθαυτό διαδικασία της μετάφρασης, η οποία αν και αρκετά δύσκολη λόγω των ειδικών όρων και τύπων, κέντρισε το ενδιαφέρον μας και μας βοήθησε στο να εμπλουτίσουμε τις γνώσεις μας. Η μετάφραση είναι κυριολεκτική, ακολουθήθηκε δε η διάταξη του ελληνικού συγγράμματος, σε μια προσπάθεια να μην ξεφύγουμε από το ύφος και το επιστημονικό περιεχόμενο του μεταφραζόμενου συγγράμματος. Εν κατακλείδι, ελπίζουμε ότι η παρούσα διπλωματική εργασία θα φανεί χρήσιμη σε φοιτητές και σε συναδέλφους μηχανολόγους μηχανικούς, καθώς επίσης και σε δυνητικούς φοιτητές ERASMUS που θα τους βοηθήσει να ασχοληθούν με τον συγκεκριμένο τομέα στην αγγλική γλώσσα.

ΒΙΒΛΙΟΓΡΑΦΙΑ

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